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MULTI-STEP CONTRIBUTIONS TO PARTICLE PRODUCTION IN NUCLEI

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Abstract: Coherent and incoherent production of particles in atomic nuclei is investigated for high energies using coupled equations of optical model form for the wave functions of the particles. We include multi-step (cascade) processes for coherent production and one incoherent step preceded or followed by coherent scattering or production for incoherent production. Calculations are performed for a coupled π, A_1, A_3 system. Given the cross sections for the corresponding reactions of pions on nucleons, we find that for coherent A_1 production the one-step process is likely dominant whereas for A_3 production two-step processes can be of considerable importance. The incoherent production cross section is changed appreciably if two-step processes are included. The unknown coupling of the A_3 to the A_1 as well as the possibility of an appreciable contribution of other intermediate boson states makes the detailed calculation of A_3 production in nuclei difficult.

1. INTRODUCTION

We consider here coherent and incoherent production of particles in atomic nuclei. This has already been studied under the assumption that the reactions consist of a pure 'one-step' process [1]. A pure one-step process consists of production of the particle being made on one nucleon preceded and followed only by processes which do not lead back to the channel under consideration.

We examine here the effect of relaxing this assumption to include the following possibilities: (i) that coherent production receives a contribution through cascade processes; (ii) that incoherent production receives a contribution from incoherent scattering or production preceded or followed by coherent production. We work in the framework of the eikonal approximation [2] and include longitudinal momentum transfer effects due to the differences in mass of the particles under consideration. As a special case we consider a coupled π, A_1, A_3 system.

2. THE OPTICAL MODEL OF PARTICLE PRODUCTION

Consider the wave equations [2]

$$[\nabla^2 + k_\alpha^2 - U_{\alpha\alpha}(r)]\psi_\alpha(r) = \sum_{\alpha' \neq \alpha} U_{\alpha\alpha'}(r)\psi_{\alpha'}(r) \ , \tag{1}$$

$$U_{\alpha\alpha'}(r) = -4\pi f_{\alpha\alpha'}(0) A \rho(r) \ . \tag{2}$$

Eq. (1) is a coupled equation of optical model form which describes the coherent production of particles α given an incident particle 1 in one of the channels α. The sum on the right side of eq. (1) is over all particles coupled to the channel α. The wave function $\psi_\alpha(r)$ describes the motion of the particle α. The $U_{\alpha\alpha'}(r)$ are functions which are proportional to the amplitude for the two-body interaction $f_{\alpha\alpha'}(0)$. This is the amplitude for producing α with particle α' incident on a nucleon at zero four-momentum transfer. We are considering processes which are diffractive in nature so that the amplitudes are taken to be independent of spin and isospin. The amplitude $f_{\alpha\alpha}(0)$ is the forward amplitude for elastic scattering of particle α by a nucleon. The $U_{\alpha\alpha'}(r)$ are also proportional to the nucleon density $A\rho(r)$ assuming the same single-particle density $\rho(r)$ for neutrons and protons; A is the total number of nucleons in the target. The quantity k_α is the magnitude of the three momentum of particle α in the lab frame. The nuclear target motion is neglected.

At high energies ($\gtrsim$ 1 GeV) the dominant production is at small angles and we are led then to consider the eikonal approximation [2]. Consider a wave with impact vector $\boldsymbol{b}$ incident in the z-direction and let us assume that the produced waves are also in the z-direction and have the same impact vector $\boldsymbol{b}$. Using standard eikonal methods to solve eq. (1) one is led to the equations

$$\frac{d\psi_\alpha(\boldsymbol{b}, z)}{dz} = ik_\alpha \psi_\alpha(\boldsymbol{b}, z) + \sum_{\alpha'} \frac{1}{2ik_\alpha} U_{\alpha\alpha'}(\boldsymbol{b}, z)\psi_{\alpha'}(\boldsymbol{b}, z) \ . \tag{3}$$

Writing

$$\psi_\alpha(\boldsymbol{b}, z) = e^{ik_\alpha z} \varphi_\alpha(\boldsymbol{b}, z) \ , \tag{4}$$

one has

$$\frac{d}{dz}\varphi_\alpha(\boldsymbol{b}, z) = \frac{1}{2ik_\alpha}\sum_{\alpha'} U_{\alpha\alpha'}(\boldsymbol{b}, z)\, e^{i(k_{\alpha'} - k_\alpha)z}\, \varphi_{\alpha'}(\boldsymbol{b}, z) \ . \tag{5}$$

3. COHERENT PROCESSES

For incident channel 1 the amplitude for outgoing channel α is

$$F_{\alpha 1} = -\sqrt{\frac{k_\alpha}{k_1}}\frac{1}{4\pi}\int e^{-i\boldsymbol{k}_\alpha\cdot \boldsymbol{r}}\, U_{\alpha\alpha'}\,\psi_{\alpha'}\, d^3r \tag{6}$$

$$= \sqrt{\frac{k_\alpha}{k_1}}\frac{k_\alpha}{2\pi i}\int e^{i\boldsymbol{q}_\alpha\cdot\boldsymbol{b}}\, d^2b[\varphi_\alpha(\boldsymbol{b}, \infty) - \delta_{\alpha 1}] \ , \tag{6a}$$

using eq. (5), where $\boldsymbol{q}_\alpha = \boldsymbol{k}_\alpha - \boldsymbol{k}_1$.

Numerical or analytical solution of eq. (5) for the $\varphi_\alpha(b,\infty)$ will therefore yield the coherent amplitudes for production and elastic scattering ($\alpha = 1$) using eq. (6a). The coherent production cross section

$$\frac{d\sigma_\alpha^{(c)}}{d\Omega} = |F_{\alpha 1}|^2 = \frac{k_\alpha^2}{\pi}\frac{d\sigma_\alpha^{(c)}}{dt} . \tag{7}$$

4. INCOHERENT PROCESSES

Let us consider the intensity $I_\alpha(b, z)$ of the wave in any channel α produced at position $(\boldsymbol{b}, z)$. Then

$$I_\alpha(\boldsymbol{b}, z) = A\rho(\boldsymbol{b}, z) \,\Big|\sum_{\alpha', \alpha''} \psi_{\alpha'}^{(1)}(\boldsymbol{b}, z) f_{\alpha''\alpha'}(q^2)\psi_{\alpha''}^{(\alpha)}(\boldsymbol{b}, -z)\Big|^2 , \tag{8}$$

where $\psi_\alpha^{(\gamma)}(\boldsymbol{b}, z)$ is the wave amplitude in channel α for an incident beam in channel γ. The sum over $\alpha'(\alpha'')$ includes all channels that can be connected coherently to channel $1(\alpha)$. This allows for any number of coherent steps (no nuclear excitation) but only one incoherent step, and hence is not valid for $e^{-aq^2} \ll 1$ where $\sqrt{a}$ is the typical range of the two-body amplitudes $f_{\alpha'\alpha''}$. In fact some processes may be dominated at all momentum transfers by multi-step incoherence. Again this expression must be corrected for correlations and other semi-coherent effects at the smallest momentum transfers. The cross section for incoherent production of α is then

$$\frac{d\sigma_\alpha^{(I)}}{d\Omega} = \int dz\, d^2b\; I_\alpha(\boldsymbol{b}, z) = \frac{k_\alpha^2}{\pi}\frac{d\sigma_\alpha^{(I)}}{dt} . \tag{9}$$

5. THE COHERENT PRODUCTION OF A_1 AND A_3 MESONS BY PIONS

We consider now the coherent production of the A_1 and A_3 mesons by incident pions. The mesons probably have spin and parity 1^+ and 2^- respectively [3] and are therefore expected to be produced diffractively on nucleons. The A_1 has been seen to be produced coherently on nuclei [4]. We assume for present purposes that the π, A_1 and A_3 form a closed system with reference to eq. (1), i.e. that coupling to other mesons are negligible. We first look at the infinite energy limit. One finds then that to good approximation eq. (1) leads to forward production amplitudes on a nucleus

$$F_{\pi A_1} = f_{\pi A_1}(0)[N_1(A, \tfrac{1}{2}\sigma) - \frac{f_{\pi A_3}(0)\, f_{A_3 A_1}(0)}{f_{\pi A_1}(0)\, f_{A_3 A_3}(0)} N_2(A, \tfrac{1}{2}\sigma)] , \tag{10}$$

$$F_{\pi A_3} = f_{\pi A_3}(0)[N_1(A, \tfrac{1}{2}\sigma) - \frac{f_{\pi A_1}(0)\, f_{A_3A_3}(0)}{f_{\pi A_3}(0)\, f_{A_1A_1}(0)} N_2(A, \tfrac{1}{2}\sigma)] , \tag{11}$$

with

$$N_m(A, \tfrac{1}{2}\sigma) = \frac{1}{m!}\frac{2}{\sigma} \int [\tfrac{1}{2}\sigma\, T(\boldsymbol{b})]^m \, e^{-\frac{1}{2}\sigma\, T(\boldsymbol{b})} \, d^2b , \tag{12}$$

where we have assumed all scattering amplitudes to be pure imaginary and have taken the total cross sections of all three bosons to be σ. The quantity

$$T(\boldsymbol{b}) = A \int_{-\infty}^{\infty} \rho(\boldsymbol{b}, z)\, dz . \tag{13}$$

Eqs. (10) and (11) include only one- and two-step contributions corresponding to the diagrams of fig. 1.

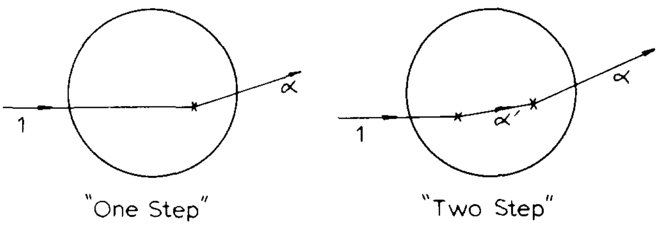


Fig. 1. Diagrams for one- and two-step production of particle α by particle 1.

Experimentally [3] it is found that $f_{\pi A_3}(0)/f_{\pi A_1}(0) \approx 0.35$ and $f_{\pi A_1}(0)/f_{\pi\pi}(0) \simeq 0.27$ at 8 GeV/c incident pion momentum. We must further know $(f_{A_1A_3}/f_{A_1A_1})$ and $(f_{A_1A_3}/f_{A_3A_3})$ in order to have a definite answer. It is reasonable to expect that these ratios will be of the same order of magnitude as $f_{\pi A_1}/f_{\pi\pi}$. For $f_{A_1A_3}/f_{A_3A_3} \cong f_{\pi A_1}/f_{\pi\pi}$ the one-step process in eq. (10), corresponding to the first term on the right side, is dominant for A_1 production, whereas for A_3 production both terms can be important. It is to be noted with respect to our conclusions for A_1 production that the ratio $f_{\pi A_1}(0)/f_{\pi\pi}(0)$ is quite large and it would be difficult to imagine that the corresponding ratios involving the A_1 and A_3 mesons would be much larger. Fig. 2 shows the values of N_1 and N_2 necessary to evaluate each term in eqs. (10) and (11). The expressions (10) and (11) can of course be generalized to look after differences in elastic amplitudes including the possibility of significant real parts.

We have a strong indication then that one can to good approximation calculate A_1 production as a one-step process. Relaxing the condition that $\sigma_{tot}(\pi n) = \sigma_{tot}(A_1 n)$ one finds [1]

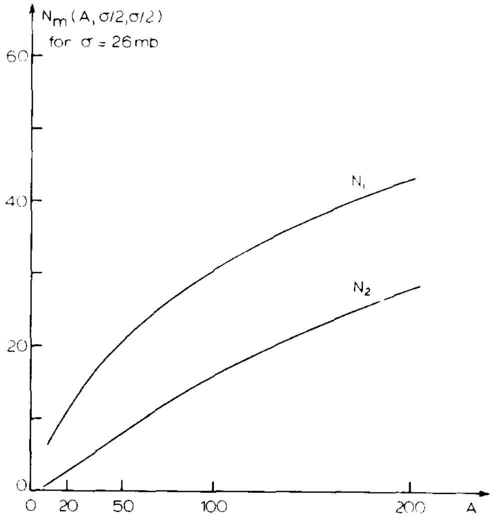


Fig. 2. $N_m(A, \frac{1}{2}\sigma)$, $m = 1, 2$ as a function of A for $\sigma = 26$ mb. The nuclear density is taken to be $\rho(r) = \rho_0/(1+\exp[(r-c)/a])$ with $a = 0.545$ fm and $c = 1.14\, A^{\frac{1}{3}}$ fm throughout this paper.

$$F_{\pi A_1}(q^2) = f_{\pi A_1}(0)\, \frac{2}{\sigma_2 - \sigma_1} \int e^{i\boldsymbol{q}\cdot\boldsymbol{b}} \left[e^{-\frac{1}{2}\sigma_1 T(\boldsymbol{b})} - e^{-\frac{1}{2}\sigma_2 T(\boldsymbol{b})}\right] d^2b \ , \qquad (14)$$

where σ_1 and σ_2 are total cross sections of π and A_1 respectively on a nucleon. This expression can then be used to determine σ_2, given reasonable data at high enough energy. The correction to this formula at finite energies will be discussed below.

It is to be noted that at finite energies the two-step process for A_1 production through the A_3 is further inhibited by longitudinal momentum transfer effects due to the higher mass of the A_3. The same would be true for possible couplings to other higher mass bosons. It is unlikely that there is any important coherent coupling to mass states lower than the A_1 (ref. [4]).

In A_3 production we are in a more complicated situation. The fact that one-step and two-step processes are comparable means that in detailed calculations we must know, or be able to determine from the production experiments, $f_{A_1A_3}/f_{A_1A_1}$ as well as $\sigma_{tot}(A_1n)$ and $\sigma_{tot}(A_3n)$. In principle we should be able to determine all of these since we have the whole periodic table to work with as targets. There is however the real possibility that other bosons contribute as intermediate states.

At finite energies we do not have simple expressions like eqs. (10) and (11) to work with. We have solved eq. (5) numerically and using eq. (6a) one finds coherent production differential cross sections on ^{64}Cu as shown in figs. 3 and 4. In these calculations all two-body amplitudes are taken to be pure imaginary. The A_3 production cross section is given for different values of the parameter

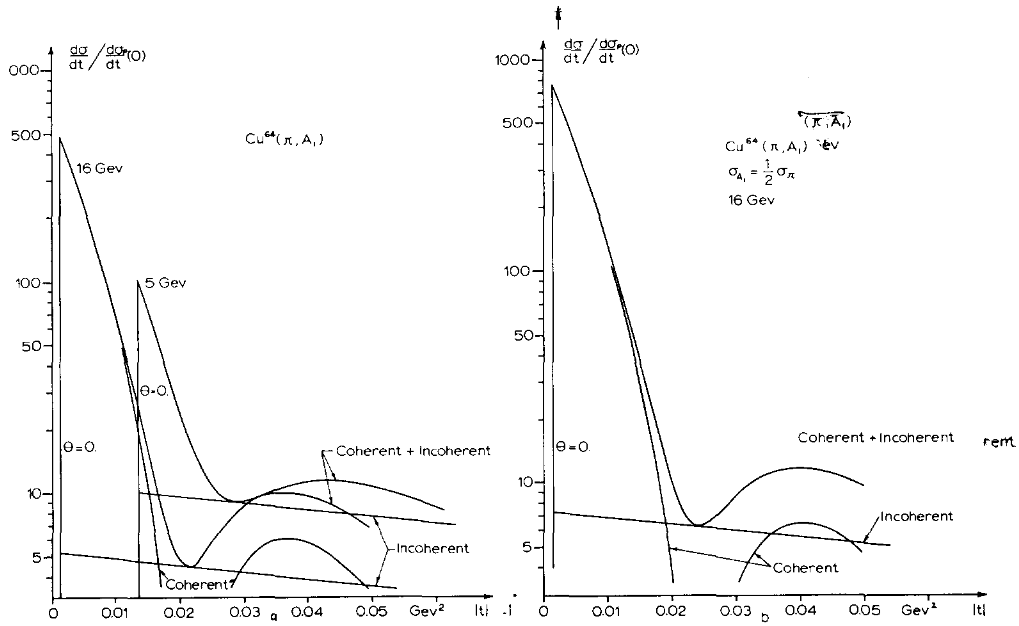
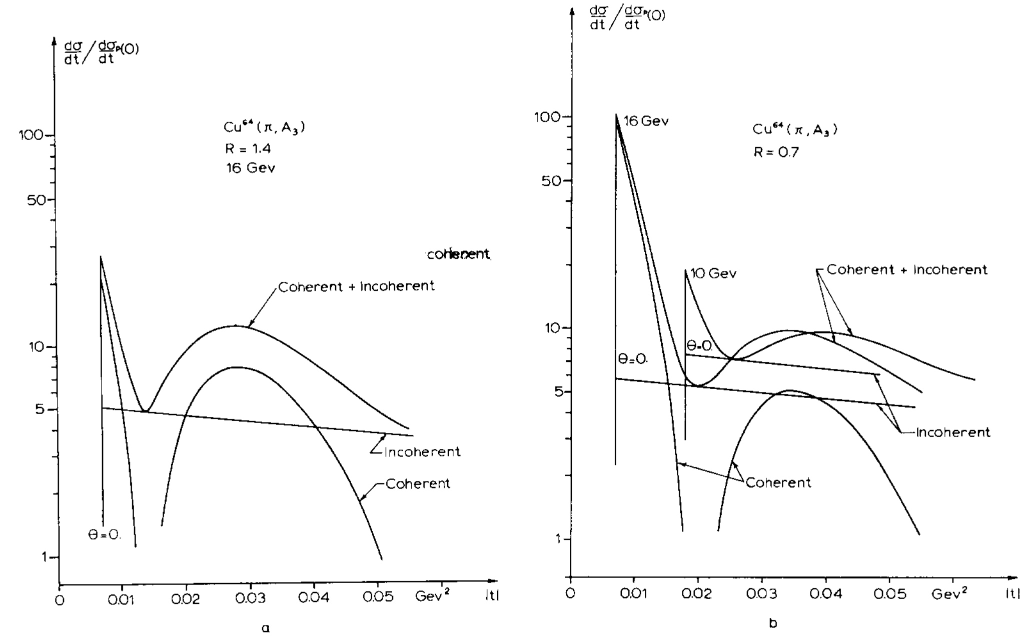


Fig. 3. Differential cross section for the A_1 production by incident pions on ^{64}Cu normalized to the cross section on protons. Calculation with the parameters $\sigma_\pi^{tot} = \sigma_{A_3}^{tot} = 26$ mb, $d\sigma/dt(\pi p \rightarrow A_1 p)_{t=0} = 2.5\ \mathrm{mb \cdot GeV^{-2}}$, $d\sigma/dt(\pi p \rightarrow A_3 p)_{t=0} = 0.3\ \mathrm{mb \cdot GeV^{-2}}$. For the calculation of the incoherent cross section an average value $A = 7.5\ \mathrm{GeV^{-2}}$ is assumed for the slopes of all differential cross sections on protons. (a) $\sigma_{A_1}^{tot} = \sigma_\pi^{tot}$; $R = 0.7$. (b) $\sigma_{A_1}^{tot} = \frac{1}{2}\sigma_\pi$; $R = 1.4$.

$$R = \frac{f_{\pi A_1}(0)\, f_{A_1 A_3}(0)}{f_{\pi A_3}(0)\, f_{A_1 A_1}(0)}\,, \qquad (15)$$

which determines the relative strength of the one-step and two-step processes. For positive values of R these two processes interfere destructively (figs. 4a and 4b). However [5] * negative values of R (fig. 4c) would occur if the production amplitude $f_{\pi A_3}(0)$ has opposite phase to $f_{\pi A_1}(0)$. For the values of R considered, A_1 production differs very little from the value obtained with $R = 0$, (no coupling of A_1 to A_3). We have taken the A_3 nucleon total cross section equal to the pion-nucleon total section, $\sigma_\pi = 26$ mb. The A_1 nucleon total cross section has been taken equal to σ_π or to $\frac{1}{2}\sigma_\pi$. (See Goldhaber et al. [1].) Table 1 lists some coherent cross sections for A_1 and A_3 production on different nuclei and for different energies. It is

* An opposite sign between $f_{\pi A_1}$ and $f_{\pi A_3}$ may be reasonable if one assumes that the A_3 production on a nucleon proceeds via two pomeron exchange. Multi-pomeron exchange has been discussed by Frautschi and Margolis and Jacob and Pokorski [5].



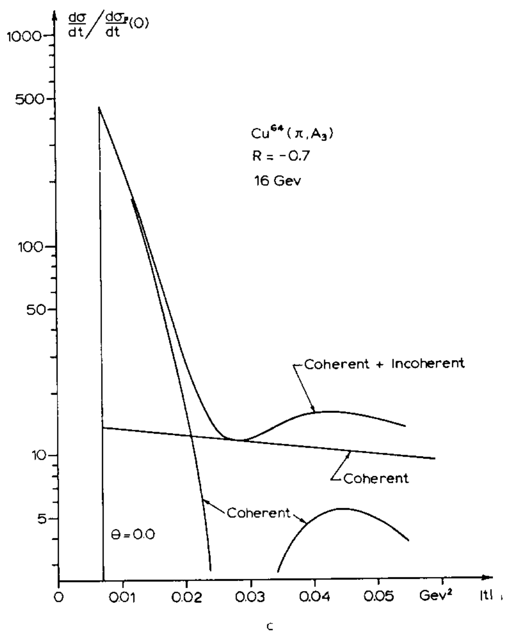


Fig. 4. Differential cross section for the A_3 production by incident pions on ^{64}Cu normalized to the cross section on protons for the parameters as listed for figs. 2 and 3 and $\sigma_{A_1}^{tot} = \sigma_{\pi}^{tot} = 26$ mb. (a) $R = 1.4$, (b) $R = 0.7$, (c) $R = -0.7$.

Table 1

Integrated coherent meson production cross section normalized to the forward differential production cross section on protons : $\sigma^{(c)}/(\mathrm{d}\sigma_p/\mathrm{d}t)(0)$ in GeV^2 for the parameters as listed for figs. 2 and 3.

		$A = 19$ 16 GeV	$A = 208$ 16 GeV	$A = 64$ ∞ energy	$A = 64$ 16 GeV	$A = 64$ 10 GeV	$A = 64$ 5 GeV	$A = 64$ 3 GeV
A_1 prod.	$\sigma_{A_1} = \sigma_\pi$, $R = 1.4$	0.94	3.4	2.5	2.2			
	$\sigma_{A_1} = \sigma_\pi$, $R = 0.7$	0.96	3.4	2.5	2.2	1.7	0.40	0.021
	$\sigma_{A_1} = \sigma_\pi$, $R = -0.7$	1.03	3.7	3.0	2.4			
	$\sigma_{A_1} = \frac{1}{2}\sigma_\pi, R = 1.4$	1.32	7.4	4.4	3.8			
A_3 prod.	$\sigma_{A_1} = \sigma_\pi$, $R = 1.4$	0.19	0.68	0.49	0.22			
	$\sigma_{A_1} = \sigma_\pi$, $R = 0.7$	0.36	0.40	1.23	0.42	0.095	0.005	0.00002
	$\sigma_{A_1} = \sigma_\pi$, $R = -0.7$	0.96	2.80	5.0	2.13			
	$\sigma_{A_1} = \frac{1}{2}\sigma_\pi, R = 1.4$	0.33	0.55	1.01	0.35			

The values of $\sigma^{(c)}$ are obtained by integrating $(\mathrm{d}\sigma^{(c)}/\mathrm{d}t)(t)$ from $t = t_{\min}$ up to $t \simeq -0.1$ GeV^2 for $A = 19$ and up to $t \simeq -0.05$ GeV^2 for $A = 64$ and $A = 208$.

to be noted that for $R \simeq -1$ at energies around 15 GeV $\sigma^{(c)}$ divided by the corresponding two-body production cross section is about equal for A_3 and A_1 production. The two-body cross section is however about ten times weaker for A_3 production than for A_1 production. For $R \approx 1$ A_3 production in the nucleus is very weak in the model studied here.

6. INCOHERENT PRODUCTION

We now discuss the calculation of incoherent production of A_1 and A_3 mesons using eqs. (8) and (9). It is to be noted [1] that the eikonal method is not expected to yield a very accurate incoherent production cross section at the smallest values of momentum transfer. However at very small values of t coherent production is dominant when it can occur.

One can write the incoherent cross section for the production of particle α by incident pions as

$$\frac{\mathrm{d}\sigma_\alpha^{(\mathrm{I})}}{\mathrm{d}\Omega} = |f_{\pi\alpha}(t)|^2 N_{\mathrm{eff}} \tag{16}$$

where N_{eff} is an effective nucleon number for the nucleus under consideration. Using only the term α' = pion and $\alpha'' = \alpha$ in the sum on the right side of eq. (8) which corresponds to one-step production of α by pions, one finds

$$N_{\mathrm{eff}} = N(A, \sigma_\pi, \sigma_\alpha) = \frac{1}{\sigma_\pi - \sigma_\alpha} \int [\mathrm{e}^{-\sigma_\alpha T(\boldsymbol{b})} - \mathrm{e}^{-\sigma_\pi T(\boldsymbol{b})}] \mathrm{d}^2 b \,, \tag{17}$$

where σ_π and σ_α are the total cross sections on a nucleon of π and α respectively. Keeping all terms of the sum in eq. (8) includes incoherent production of particle α by a particle α' that itself is produced coherently (terms $\alpha'' = \alpha$, $\alpha' \neq \alpha$), coherent production of α followed by incoherent scattering of α (term $\alpha'' = \alpha' = \alpha$) and coherent production of α preceded by an incoherent process $\alpha' \to \alpha''$ ($\alpha \neq \alpha''$). The contribution to the incoherent production of the multi-step processes can be substantial. Some values of N_{eff} are given in table 2 for different values of the parameters of the theory. Figs. 3 and 4 show incoherent as well as coherent production of A_1 and A_3 and the sum of coherent and incoherent cross sections with the parameters listed.

Table 2
Effective number N_{eff} for incoherent meson production for the parameters as listed for figs. 2 and 3.

	$A = 19$ 16 GeV	$A = 208$ 16 GeV	$A = 64$ ∞ energy	$A = 64$ 16 GeV	$A = 64$ 10 GeV	$A = 64$ 5 GeV	$A = 64$ 3 GeV
'one step'	7.0	16.0	11.5	11.5	11.5	11.5	11.5
'*multi step*': pion incoh. scat.	6.8	15.6	11.0	11.1	11.2	11.5	11.5
A_1 prod. $\sigma_{A_1} = \sigma_\pi$; $R = 1.4$	3.3	8.8	4.0	5.1			11.5
A_1 prod. $\sigma_{A_1} = \sigma_\pi$; $R = 0.7$	3.3	8.9	4.2	5.2	6.6	10.2	11.5
A_1 prod. $\sigma_{A_1} = \sigma_\pi$; $R = -0.7$	3.6	9.6	4.4	5.6			11.5
A_1 prod. $\sigma_{A_1} = \frac{1}{2}\sigma_\pi$; $R = 1.4$	4.6	12.4	5.5	7.2			16.7
A_3 prod. $\sigma_{A_1} = \sigma_\pi$; $R = 1.4$	2.8	8.2	4.8	5.2			11.8
A_3 prod. $\sigma_{A_1} = \sigma_\pi$; $R = 0.7$	3.4	8.5	3.9	5.8	7.6	10.8	11.8
A_3 prod. $\sigma_{A_1} = \sigma_\pi$; $R = -0.7$	7.0	2.3	6.3	13.6			11.9
A_3 prod. $\sigma_{A_1} = \frac{1}{2}\sigma_\pi$; $R = 1.4$	3.4	9.4	4.7	6.1			11.9

7. DISCUSSION

It can be seen from the above that the simple one-step theory which one is familiar with for coherent and incoherent diffractive production can be in serious error. In the case of a relatively light and strongly produced boson like the A_1 meson the one-step theory should be adequate for coherent production, for the A_3 meson it is likely not. In incoherent production at energies of the order of 10 GeV or greater the simple one-step theory needs corrections due to coherent production followed by incoherent scattering. By going to lower energies this correction may become negligible due to longitudinal momentum-transfer considerations. However since the details of the coupling strength of one unstable boson to another are unknown, it is difficult to evaluate the corrections in detail. To this end experiments on coherent and incoherent production of pion (and kaon) resonances will be very valuable. With enough data perhaps one will be able to sort out coupling strengths as well as unstable particle cross sections. If this is the case, the study of high-energy reactions on nuclei will yield an even larger amount of information on the unstable particles than one has imagined. The other possibility is of course that one may be able to study the lower lying unstable particles profitably using nuclear targets.

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