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REGGE CUTS AND NEUTRAL PION PHOTOPRODUCTION *

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Abstract: Photoproduction of neutral pions on protons is analyzed in terms of models involving exchange of the ω Regge pole and Regge cuts due to ω pole plus the pomeron. The differential cross section in a wide range of energies and the asymmetry ratio are well accounted with a small number of free parameters.

1. INTRODUCTION

One of the most important cases in favour of Regge cuts is the photoproduction of neutral pions or protons. In Regge pole analysis this is dominated by exchange of the ω trajectory $\alpha_\omega(t)$. Standard reggeization implies that the differential cross section must show a dip at $\alpha_\omega = 0$; this is indeed observed at $t \approx -0.6$ GeV^2 at relatively low photon lab. momenta ($k_\gamma \lesssim 6$ GeV) (refs. [1, 2]). However, at higher energy this dip completely disappears [2]. Simple Regge pole models, consisting of ω plus some lower-lying trajectory, predict a dip more pronounced with increasing energy. Thus we should seek a different picture.

We consider models consisting of the ω Regge pole plus a series of complex angular momentum (J) branch points $\alpha_n(t)$ formed by exchange of ω plus n pomerons. The branch point due to exchange of ω plus one pomeron of trajectory $\alpha_\mathrm{P}(t)$ is given by [3]

$$\alpha_1(t) = \max\{\alpha_\omega(t') + \alpha_\mathrm{P}(t'') - 1\} , \tag{1.1}$$

where t', $t'' \leq 0$ and $(-t')^{\frac{1}{2}} + (-t'')^{\frac{1}{2}} \leq (-t)^{\frac{1}{2}}$ $(t \leq 0)$; etc. In the linear trajectory approximation

$$\alpha_\omega(t) = \alpha_\omega^0 + \lambda_\omega t , \qquad \alpha_\mathrm{P}(t) = 1 + \lambda_\mathrm{P} t , \tag{1.2}$$

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which is expected to be valid in the region of interest ($0 < -t \lesssim 1.2$), Eq. (1.1) is easily seen to lead to

$$\alpha_n(t) = \alpha_\omega^0 + \lambda_n t \, , \qquad \lambda_n^{-1} = \lambda_\omega^{-1} + n\lambda_P^{-1} \, . \tag{1.3}$$

Thus, e.g. for $t \cong -0.6$: $\alpha_n(t) > \alpha_\omega(t)$. This implies that with increasing energy the Regge cuts dominate over the ω pole. With the dip at $t \approx -0.6$ associated with zeros in the ω residues and with the cuts contributing a smooth, non-vanishing function of t, it is easily seen that at sufficiently high energy the dip must disappear.

2. STRUCTURE OF THE MODEL

Since in our approach the dip is related to nonsense factors, it is simpler to proceed via t-channel helicity amplitudes $f_{\lambda_\gamma\lambda_\pi,\lambda_N\lambda_{\bar{N}}}(t, \cos\theta_t)$. Following standard procedures [4], we define parity conserving ones $\bar{f}^\sigma_{\lambda\mu}$ where $\lambda = \lambda_\gamma - \lambda_\pi$, $\mu = \lambda_N - \lambda_{\bar{N}}$ and $\sigma = +(-)$ denotes natural (unnatural) parity. The ω pole, which provides the driving force, contributes to $\bar{f}^+_{01}$ and $\bar{f}^+_{11}$ with

$$K_\lambda^{-1}(t)\,\bar{f}^{+(\omega)}_{\lambda 1}(s,t) \equiv f^{+(\omega)}_{\lambda}(s,t) = b_\lambda(t)\,(\mathrm{e}^{-\frac{1}{2}i\pi}\,s)^{\alpha_\omega(t)-1} \, , \tag{2.1}$$

where the energy scale has been chosen, as usual, $s_0 = 1\ \mathrm{GeV}^2$ and

$$K_0(t) \equiv t - \mu^2 \, , \qquad K_1(t) \equiv t^{-\frac{1}{2}}(t - \mu^2) \, ,$$

(μ = pion mass). With the kinematic factors $K_\lambda(t)$ taken out, $b_\lambda(t)$ are analytic functions in the t-plane cut along $9\mu^2 \leqslant t < \infty$.

The functions $b_\lambda(t)$ vanish when $\alpha_\omega(t)$ crosses the integers $J = 0, -2, \ldots$; they may vanish also at $J = -1, -3, \ldots$ We shall take

$$b_0(t) = \gamma_0(\alpha^0 + \lambda_\omega t)(1 + \alpha_\omega^0 + \lambda_\omega t)(2 + \alpha_\omega^0 + \lambda_\omega t)\ldots \tag{2.2a}$$

(γ_0 = const.) Analyticity and factorization requirements for the ω pole [5] imply $b_1 \sim t$ as $t \to 0$, so that

$$b_1(t) = \gamma_1 t(\alpha^0 + \lambda_\omega t)(1 + \alpha_\omega^0 + \lambda_\omega t)(2 + \alpha_\omega^0 + \lambda_\omega t) \, . \tag{2.2b}$$

The constants γ_0 and γ_1 are, in general, free. However, the ratio γ_1/γ_0 can roughly be estimated from Regge pole analysis of NN $\to$ NN, where ω contributes significantly; and ref. [6] gives $1 \lessapprox \gamma_1/\gamma_0 \lessapprox 3$. Then assuming vector dominance, we can calculate, say, γ_0 from the ω-exchange contribution to πN $\to$ ρN [7].

To calculate the cut contribution to $\bar{f}^\sigma_{\lambda\mu}$ notice first that for $|t| \lesssim 1$ the amplitude for pomeron exchange can well be written neglecting spin:

$$f^{(P)}(s,t) = \xi\,(e^{-\frac{1}{2}i\pi}s)^{\alpha_P(t)} \qquad \xi = \text{real const.} \,. \tag{2.3}$$

Next we use methods developed in Regge cut models for elastic N-N and π-N scattering [8,9]. For (2.3) we introduce the Hankel transform:

$$F^{(P)}(b,s) = \int_0^\infty f^{(P)}(s,-q^2)\, J_0(bq)\, q\, dq \tag{2.4}$$

($t = -q^2$) and for (2.1):

$$F_\lambda^{(\omega)}(b,s) = \int_0^\infty f_\lambda^{+(\omega)}(s,-q^2)\, J_0(bq)\, q\, dq\,. \tag{2.5}$$

The contribution $f_\lambda^{c_1}$ of the first cut ($\omega + P$) will be given by the Hankel transform

$$f_\lambda^{+c_1}(s,t) = 2\lambda_P \int_0^\infty F_\lambda^{(\omega)}(b,s)\, F^{(P)}(b,s)\, J_0(bq)\, b\, db\,. \tag{2.6}$$

On the other hand, by writing

$$f_\lambda^{+\text{cuts}}(s,t) = \int_0^\infty F_\lambda^{(\omega)}(b,s)\, (e^{2\lambda_P F^{(P)}} - 1)\, J_0(bq)\, b\, db\,. \tag{2.7}$$

and expanding

$$e^z - 1 = \sum_{n=1}^\infty \frac{1}{n!} z^n$$

we construct an infinite series of cuts (in accord with models of field theory or s-channel unitary iterations of Regge poles). As we shall show, these cuts have all the properties established in explicit dynamical models.

In $\gamma N \rightarrow \pi N$ it is well known that, at $t = 0$, $\bar{f}_{11}^{+}$ and $\bar{f}_{01}^{-}$ satisfy a kinematical constraint (conspiracy relation). Introducing

$$f_0^-(s,t) \equiv t^{\frac{1}{2}}(t - 4M^2)^{-\frac{1}{2}}\,(t - \mu^2)^{-1}\,\bar{f}_{01}^-(s,t)\,, \tag{2.8}$$

we reduce this constraint in the form

$$f_0^-(s,0) = (2M)^{-1}\, f_1^+(s,0)\,. \tag{2.9}$$

In contrast to a Regge pole which asymptotically contributes to helicity amplitudes with definite σ (= + or -) a Regge cut contributes to both σ = + and σ = -. In view of (2.2b), $f_1^{+(\omega)}(s,0) = 0$, so that (2.9) requires

$$f_0^-(s,0) = (2M)^{-1}\, f_1^{+\text{cuts}}(s,0)\,. \tag{2.10}$$

This relation introduces some unnatural (σ = -) parity exchange without extra parameters.

3. REGGE CUT CONTRIBUTIONS

In phenomenological applications for $-t \lesssim 1\ \mathrm{GeV}^2$ the ω Regge pole contribution can well be approximated by keeping only the first parenthesis $(\alpha_0^0 + \lambda_\omega t)$ in (2.2). At very high energy ($k_\gamma \gtrsim 15$ GeV) the same holds for the cut contributions constructed as in (2.6) or (2.7). Here, however, we want to account for detailed experimental information (including data with polarized photons) which extends to rather low energies ($2.8 \leqslant k_\gamma \leqslant 16$). Even down to $k_\gamma = 2.8$ we find that $f_0^{+(\mathrm{cuts})}(s,t)$ is not sensitive to the exact form of (2.2a); and that it is sufficient to use:

$$F_0^{(\omega)}(b,s) = \gamma_0 \int_0^\infty (\alpha_\omega^0 - \lambda_\omega q^2)\,(\mathrm{e}^{-\frac{1}{2}i\pi} s)^{\alpha_\omega(-q^2)-1} J_0(bq)\, q\, \mathrm{d}q\,. \tag{3.1a}$$

However, for $k_\gamma \lesssim 15$, $f_1^{+(\mathrm{cuts})}(s,t)$ is sensitive to the exact form of (2.2b). Thus we shall use the following simple parametrization

$$F_1^{(\omega)}(b,s) = -\gamma_1 \int_0^\infty q^2(\alpha_\omega^0 - \Lambda q^2)\,(\mathrm{e}^{-\frac{1}{2}i\pi} s)^{\alpha_\omega(-q^2)-1} J_0(bq)\, q\, \mathrm{d}q \tag{3.1b}$$

where Λ = free parameter.

The Hankel transforms (3.1) can be calculated explicitly to give [10]:

$$F_0^{(\omega)}(b,s) = \gamma_0\,(\mathrm{e}^{-\frac{1}{2}i\pi} s)^{\alpha_\omega - 1}\, 2\theta_\omega^{-2}\, \mathrm{e}^{-b^2/\theta_\omega^2} \{\alpha_\omega^0 - \frac{1}{\rho} L_1\,(b^2/\theta_\omega^2)\}\,, \tag{3.2a}$$

$$F_1^{(\omega)}(b,s) = -\gamma_1\,(\mathrm{e}^{-\frac{1}{2}i\pi} s)^{\alpha_\omega - 1}\,(\tfrac{1}{2}\theta_\omega)^{-4}\, \mathrm{e}^{-b^2/\theta_\omega^2}$$
$$\{\tfrac{1}{2}\alpha_\omega^0\, L_1\,(b^2/\theta_\omega^2) - \Lambda\,(\tfrac{1}{2}\theta_\omega)^{-2}\, L_2\,(b^2/\theta_\omega^2)\}\,, \tag{3.2b}$$

where

$$\rho \equiv \ln s - \tfrac{1}{2}i\pi\,, \qquad \theta_\omega^2 \equiv 4\rho\lambda_\omega \tag{3.3}$$

and $L_m(b^2/\theta_\omega^2)$, $m = 1, 2$, are the Laguerre functions of order m.

We shall proceed with the general model of an infinite series of cuts. Replacing (3.2) in (2.7) and using the transform [10]

$$\int_0^\infty b\, \mathrm{d}b\, J_0(bq)\, \mathrm{e}^{-\beta b^2}\, L_n(\alpha b^2) = \frac{(\beta-\alpha)^n}{2\beta^{n+1}}\, \mathrm{e}^{-q^2/4\beta}\, L_n\left(\frac{\alpha q^2}{4\beta(\alpha-\beta)}\right)\,,$$

we obtain

$$f_0^{+(\mathrm{cuts})}(s,t) = \gamma_0 \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\xi}{\rho}\right)^n \frac{\lambda_n}{\lambda_\omega} \{\alpha_\omega^0 - \frac{n\lambda_n}{\rho\lambda_P} L_1(z_n)\} (\mathrm{e}^{-\frac{1}{2}i\pi} s)^{\alpha_n(t)-1} , \quad (3.4\mathrm{a})$$

$$f_1^{+(\mathrm{cuts})}(s,t) = -\gamma_1 \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\xi}{\rho}\right)^n \left(\frac{\lambda_n}{\lambda_\omega}\right)^2 \frac{n}{\rho\lambda_P}$$

$$\{\alpha_\omega^0 L_1(z_n) - \frac{2n\lambda_n\Lambda}{\rho\lambda_P\lambda_\omega} L_2(z_n)\} (\mathrm{e}^{-\frac{1}{2}i\pi} s)^{\alpha_n(t)-1} , \quad (3.4\mathrm{b})$$

where $\alpha_n(t)$ as in (1.3) and

$$z_n \equiv t\lambda_n \rho \frac{\lambda_P}{n\lambda_\omega} . \quad (3.5)$$

For $|t| \lesssim 1$ and the energies of interest, $|z_n| \ll 1$ and (3.4) can be simplified by taking $L_m(z_n) \approx 1$.

In the linear trajectory approximation the interpretation of the various terms in the series (3.4a) via singularities in complex J is straightforward: First, the exponent $\alpha_n(t)$ gives exactly the position of a moving branch point due to exchange of ω plus n pomerons. Also, the factor $\exp(-\frac{1}{2}i\pi\ \alpha_n(t)$ correctly establishes the asymptotic phase of the corresponding cut [11]. Finally, it is known that as the number of the exchanged pomerons increases, the cuts contain decreasing powers of $\ln s - \frac{1}{2}i\pi$. In view of $L_1(z_n) = 1 + z_n$, to each order n the terms

$$\alpha_\omega^0 - \frac{n\lambda_n}{\rho\lambda_P} L_1(z_n) = \alpha_\omega^0 - \frac{n\lambda_n}{\rho\lambda_P} - \frac{\lambda_n^2}{\lambda_\omega} t \quad (3.6)$$

can be interpreted as superposition of 3 cuts with different discontinuities near the branch point $J = \alpha_n(t)$. Clearly, a similar interpretation holds for (3.4b).

In the series (3.4) as well as in similar expansions in NN → NN and πN → πN (refs. [8,9]) the power n of the real parameter ξ equals the number of pomerons contributing to the branch point at $J = \alpha_n(t)$; thus it is reasonable to consider ξ as an effective average coupling of P to the scattered particles. Then ξ can be estimated by constructing a similar model for $\gamma p \to \pi^+ n$ at small $|t|$, when Regge cuts are also expected to be very important. Also, in the vector dominance model, $\gamma p \to \pi^0 p$ (ω exchanged) and forward $\gamma p \to \pi^+ n$ (π^+ exchanged) are related to $\rho N \to \pi N$; thus the pomeron is coupled to the same external particles and is expected to have comparable strength.

4. NUMERICAL APPLICATIONS AND DISCUSSION

We shall analyze the data for $\gamma p \to \pi^0 p$ using two different Regge cut models. In both of them the ω pole contribution is taken

$$\bar{f}^{+(\omega)}_{\lambda_1}(s,t) = K_\lambda(t)\beta_\lambda(t)\alpha_\omega(t)\,(e^{-\frac{1}{2}i\pi}s)^{\alpha_\omega(t)-1} \tag{4.1}$$

with

$$\beta_0 = \gamma_0\,, \qquad \beta_1(t) = \gamma_1 t\,; \tag{4.2}$$

in accord with most phenomenological applications use

$$\alpha_\omega(t) = 0.475 + 0.86\,t\,, \qquad (\alpha_\omega(t \simeq -0.55) = 0)\,.$$

Each $\bar{f}^{+}_{\lambda_1}(s,t)$ will in addition contain a cut contribution $K_\lambda(t)\, f^{+(\mathrm{cuts})}_\lambda(s,t)$

In the first model $f^{+(\mathrm{cuts})}_\lambda$ will be constructed by taking in (3.4) only one cut (the term $n = 1$), which thus represents the overall series in an average sense; for relatively small $|t|$ this is a usal approach. Then, we shall fix $\lambda_P = 0.3\ \mathrm{GeV}^{-2}$ and, as usual in one-cut models, take $\rho \to \ln s$ (asymptotic one-cut form). With $L_m(z_1) \approx 1$, (3.4) give:

$$f^{+(\mathrm{cut})}_0(s,t) = \gamma_0 \frac{\xi}{\ln s}\frac{\lambda_1}{\lambda_\omega}\left(\alpha^0_\omega - \frac{\lambda_1}{\lambda_P \ln s}\right)(e^{-\frac{1}{2}i\pi}s)^{\alpha_1(t)-1}\,, \tag{4.3a}$$

$$f^{+(\mathrm{cut})}_1(s,t) = -\gamma_1 \frac{\xi}{\lambda_P(\ln s)^2}\left(\frac{\lambda_1}{\lambda_\omega}\right)^2\left(\alpha^0_\omega - \frac{2\lambda_1\Lambda}{\lambda_P\lambda_\omega \ln s}\right)(e^{-\frac{1}{2}i\pi}s)^{\alpha_1(t)-1}\,. \tag{4.3b}$$

The value of ξ has been determined through the equivalent one-cut model of $\gamma p \to \pi^+ n$ [12]; we find

$$\xi = -8.1\,. \tag{4.4}$$

We take $\gamma_1/\gamma_0 = 1.5$. Finally, we fix $\Lambda = \frac{1}{2}\lambda_\omega$, which gives $f^{+(\mathrm{cut})}_1(k_\gamma, t)$ smoothly varying for all $k_\gamma \gtrsim 2$ GeV/c. For t sufficiently small the constraint (2.10) implies that $f^{-}_0(s,t)$ also receives a contribution

$$f^{-}_0(s,t) \simeq (2M)^{-1} f^{+(\mathrm{cut})}_1(s,t)\,. \tag{4.5}$$

We shall assume (4.5) for all $-t \lesssim 1\ \mathrm{GeV}^2$; and this is very important in calculating the assymetry

$$R = \frac{\mathrm{d}\sigma_\perp/\mathrm{d}t - \mathrm{d}\sigma_\parallel/\mathrm{d}t}{\mathrm{d}\sigma_\perp/\mathrm{d}t + \mathrm{d}\sigma_\parallel/\mathrm{d}t}\,, \tag{4.6}$$

where $\sigma_\perp(\sigma_\parallel)$ the cross section for photons polarized perpendicular (paral-

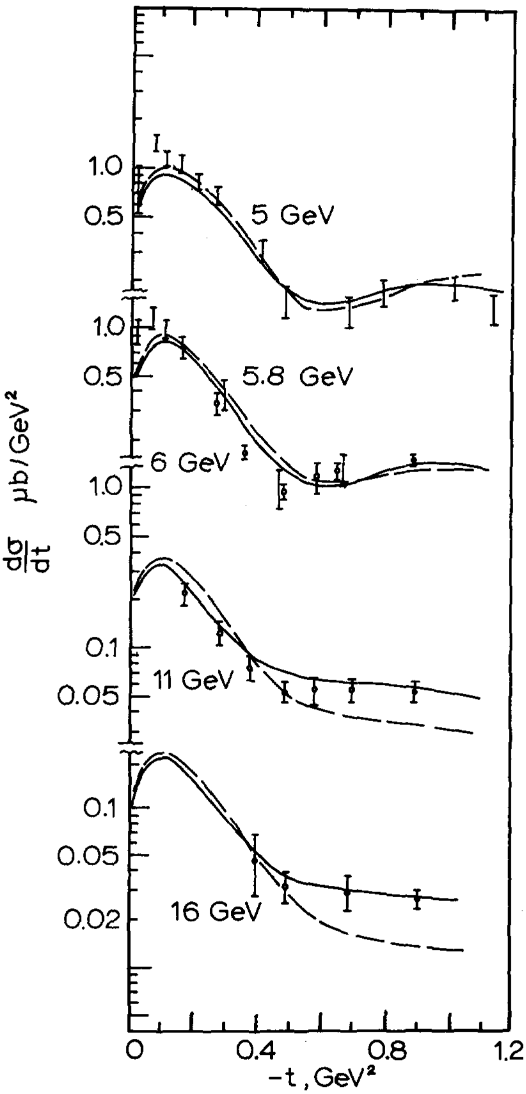

FIG. 1

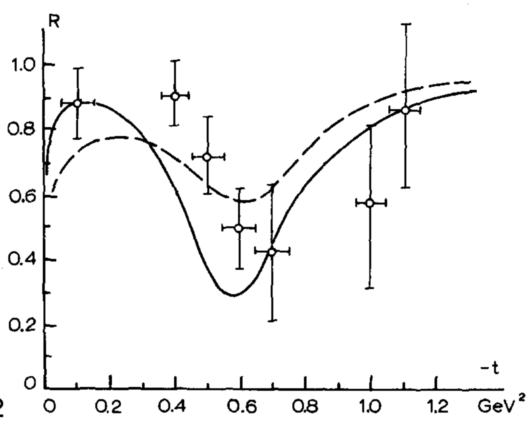

FIG. 2

Fig. 1. Calculated differential cross sections at lab. momenta $k_\gamma = 5$, 6, 11 and 16 GeV. Full lines: one-cut model; dashed lines: model with infinite series of cuts. Data: I from ref.[1]; Φ from ref.[2].

lel) to the production plane. The differential cross section $d\sigma/dt$ ($= d\sigma_{\perp}/dt + d\sigma_{\parallel}/dt$) is calculated from

$$\tfrac{1}{2}\pi\,(s - M^2)^2 \frac{d\sigma}{dt} = \sum_{\lambda_\gamma \lambda_N \lambda_{\overline{N}}} |f_{\lambda_\gamma \rho, \lambda_N \lambda_{\overline{N}}}|^2 \tag{4.7}$$

and the results are shown in figs. 1 and 2 (solid lines).

The second cut model keeps the complete series in (3.4) with exact factors $\rho = \ln s - \frac{1}{2}i\pi$. We take $\Lambda = \frac{1}{2}\lambda_\omega$, as before, and $\lambda_P = 0.8$. The equivalent infinite cut model applied to $\gamma p \to \pi^+ n$ leads to excellent agreement with [12]

$$\xi = -10.5\ . \tag{4.8}$$

Here the value $\gamma_1/\gamma_0 = 2.5$ leads to fair agreement (figs. 1 and 2; dashed lines).

The physical aspects, which are more or less in common to both our models, can be summarized as follows: At relatively low energy ($k_\gamma \lesssim 10$) the pole contributions $\bar{f}^{+(\omega)}_{\lambda 1}(s,t)$ control the behaviour of $d\sigma/dt$; thus, at $t \simeq -0.55$ we have a dip. Also, $\sigma_\perp$ contains only exchanges with $\sigma = +$ (i.e. f_0^+ and f_1^+) and $\sigma_\parallel$ only $\sigma = -$ (i.e. f_0^-); away from the dip the pole contributions enhance $d\sigma_\perp/dt$, so the asymmetry R is rather large. At $t \simeq -0.55$, R also has a dip; there is, however, still significant contribution to $\bar{f}^+_{01}$ and f^+_{11} from the cuts, thus leading to $R > 0$ event at the dip.

At higher energy the relative importance of the cuts increases, as can most easily be seen in the one-cut model (4.3): For $2.8 \lesssim k_\gamma \lesssim 16$ GeV the quantity

Fig. 2. The asymmetry ratio R at $k_\gamma = 2.8$ GeV. Full line: one-cut model; dashed line: model with infinite series of cuts. Data: Bellenger et al., Proceedings of the XIV Intern. Conference on H.E. Physics (Vienna, 1968) p. 3.

$$\frac{1}{\ln s}\left(\alpha_\omega^{\,o} - \frac{\lambda_1}{\lambda_P \ln s}\right)$$

remains essentially constant, whereas, with $\Lambda = \frac{1}{2}\lambda_\omega$,

$$\frac{1}{(\ln s)^2}\left(\alpha_\omega^{\,o} - \frac{\lambda_1}{\lambda_P \ln s}\right)$$

decreases slowly. Thus the Regge cuts gradually take over and the dip is washed out.

It is of interest that our values of the parameter ξ (in (4.4) and (4.8)) are nearly the same as those describing elastic scattering in equivalent models (e.g. the second of ref. [9] gives $\xi = -7$ for pp $\rightarrow$ pp and $\xi = -10$ for $\bar{p}p \rightarrow \bar{p}p$). This further supports the interpretation of ξ as an average pomeron "coupling" and allows a significant reduction of the number of free parameters in Regge cut models of two-body inelastic reactions.

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