Blocking Model for All-Optical Overlaid-Star TDM Networks

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***Abstract*—This paper studies the blocking performance of a class of all-optical overlaid-star TDM networks using a least-congested-path routing strategy in path selection. An analytical model is proposed to estimate the call blocking probability in such networks. This model takes link-load correlation into account and thus can provide accurate estimation of the blocking performance. The accuracy of the analytical model is verified by comparing analytical results with simulation results.**

I. INTRODUCTION

We study a class of all-optical networks that employ an overlaid-star topology and use time division multiplexing (TDM) for data transmission, as shown in Figure 1. This class of networks features the ability to dynamically allocate bandwidth on demand at a fine granularity, and the concentration of control and routing functionality at the electronic edge nodes that surround the photonic core [1]. The overlaid-star topology provides robustness in the case of a network failure, such as a fiber cut or a device fault, and also relieves potential network congestion. In such networks, a connection is established on a path between a pair of edge nodes and is allocated one or multiple timeslots on a wavelength. If no timeslot is available for a connection request or call, the call will be blocked. Hence, call blocking probability is a primary performance metric in such networks. In general, routing has a big impact on the blocking performance. The shortest path routing cannot achieve best performance [2]. To improve network performance, the network state should be taken into account in routing. On the other hand, the blocking performance is also subject to various network constraints, such as the wavelength continuity constraint [2] and the constraint in the case of no timeslot interchangers (TSIs) available in the core switches. A TSI allows a switch to rearrange the order of the timeslots on its input and output ports, which would largely improve the blocking performance. In an all-optical TDM switch, TSIs can be implemented by using fiber delay lines (FDLs). However, this would introduce additional propagation delay and largely increase the cost and complexity of the switch.

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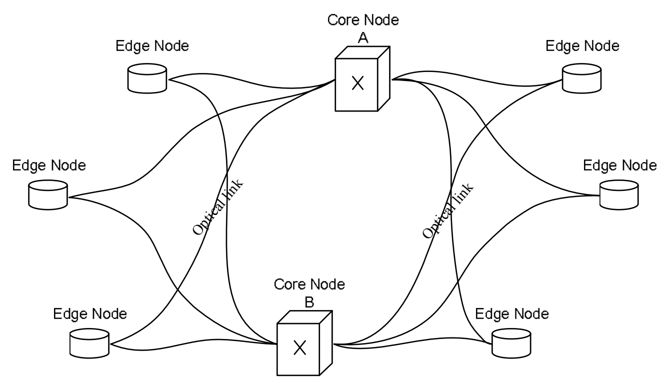

Figure 1 Overlaid-star topology

In this paper, we consider a least-congested-path routing strategy for path selection. With this routing strategy, the source node of a call selects the least-congested path to the destination, making the traffic load on each link more balanced and network performance improved. An analytical model is proposed to estimate the blocking performance of the network. This analytical model takes into account the link-load correlation between two consecutive links and thus can provide accurate estimation of the blocking performance.

II. NETWORK ACHITECTURE AND RELEATED WORK

In this section, we briefly describe the network architecture and review related work.

1. Network Architecture

The all-optical overlaid-star TDM network architecture consists of edge nodes interconnected via several central core nodes in an overlaid star topology, as shown in Figure 1. Each edge node is connected to a core node by a couple of fibers, one for transmission in each direction. An edge node is a hybrid electro-optical component that serves as an interface between the optical network and an electronic outside network, such as an IP network or SONET network. A core node employs an all-optical space switch that can switch an input wavelength on an input port to an output port,

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making data paths inside the core node purely optical and transparent.

The network uses TDM for data transmission. In TDM, a wavelength is divided into a series of frames, each consisting of a fixed number of timeslots. Each core switch has an associated electronic controller that performs timeslot allocation, switch configuration, and other control functions. To establish a connection, the source node, destination node, and core node participate in a signaling protocol to allocate timeslots for the connection. The control messages are exchanged between edge nodes and core nodes out of band over a dedicated control timeslot (or channel) on each wavelength. There is one control timeslot per frame in either direction. No wavelength converters and TSIs are available in each core switch.

2. Related Work

Blocking performance has been widely studied for all-optical WDM networks. A variety of analytical models have been proposed to compute the call blocking probability with different network constraints, traffic models, and routing and wavelength assignment algorithms [3-9]. For example, Kovacevic et al. proposed an approximate analytical model in [3] to compute the blocking probability of an all-optical network both with and without wavelength conversion. This model, however, only considers static routing and does not consider the load correlation between successive links of a path. In [4], Barry et al. proposed an analytical model to compute the blocking probability of a multi-hop path in all-optical networks, taking wavelength correlation into account. This model makes more simplistic traffic assumptions and does not take into account the dynamic nature of network traffic. In [5], Subramaniam et al. proposed an analytical model that takes both dynamic traffic and link-load correlation into account and has a moderate complexity. In [6], Li et al. proposed an approximate analytical model to analyze the blocking performance of fixed-path least congestion routing and dynamic routing using neighborhood information with link-load correlation considered.

In the context of all-optical TDM networks, Yates et al. analyzed the blocking performance of multi-wavelength TDM networks based on both analytical and simulation results [7]. In [8], Sivakumar et al. investigated the effects of wavelength conversion and timeslot interchange on the blocking performance of TDM wavelength routing networks based only on simulation experiments. In [9], Wen et al. analyzed the blocking performance of a family of wavelength and timeslot assignment algorithms for TDM wavelength-routed networks based only on simulation results.

III. Analytical Model for Least-Congested-Path Routing

In this section, we first describe a least-congested-path routing strategy and then present an analytical model to compute the blocking performance with the routing strategy.

1. Network Model

The overlaid-star network can be modeled as a three dimensional directed graph $G = (V_1, V_2, E)$, where V_1 and V_2 denotes two different sets of vertices and E stands for the set of directed edges. Each vertex v_i $(i = 1, \cdots, N)$ in V_1 corresponds to an edge node and each vertex v_k (or c_k) $(k = 1, \cdots, M)$ in V_2 corresponds to a core node, where N is the number of edge nodes and M is the number of core nodes in the network. Each edge e_{ik} (or e_{ki}) in E corresponds to a fiber link between edge node i and core node k (or between core node k and edge node i). Each fiber link has one wavelength with L timeslots in each frame, which are denoted by $S = \{s_1, s_2, \cdots, s_L\}$. There exist a fixed set of M two-hop paths between a pair of edge nodes, each passing through one of the core nodes, as shown in Figure 2. The set of paths between edge node i and edge node j can be denoted by $P_{ij} = \{p_{ij}^k; i, j = 1, \cdots, N; k = 1, \cdots, M\}$, where $p_{ij}^k = \{e_{ik}, e_{kj}\}$ denotes the path passing through core node k and consisting of edge e_{ik} and edge e_{kj}.

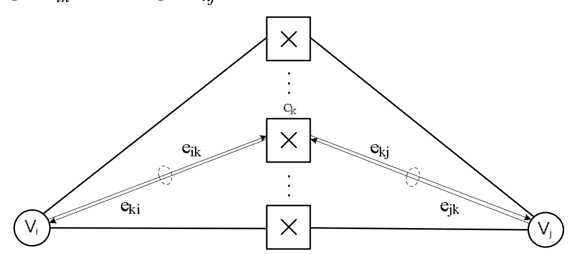

Figure 2 A set of two-hop paths between a pair of edge nodes

2. Least-Congested-Path Routing

As mentioned in Section 2.1, there exist a fixed set of M candidate paths between a pair of edge nodes, each passing through one of the core nodes, as shown in Figure 2. To accommodate a call, the source node must first select one of the M candidate paths to the destination. The traditional shortest-path routing strategy may result in a situation where some of the links are overloaded while the other links are underutilized, which would largely increase the blocking probability in the network. To achieve good performance, a routing strategy should be able to make a path selection based on the current timeslot usage on each link. For each call, the source node should select the least congested path to the destination, thus making the timeslot usage on each link more balanced and the blocking probability reduced. Here the congestion of a path is measured in terms of the number of timeslots available on the most congested link of the path. The congestion of a link is measured in terms of the number of timeslots available on the link. The less the number of available timeslots, the more congested the link. This strategy is called the least-congested path strategy. To support this routing strategy, each edge node must maintain the state information on each link. This information is advertised and updated by each core node periodically using a signaling protocol. The link state information contains the timeslot usage in each frame.

3. *Analytical Model*

The assumptions made in the analytical model are given as follows:

- Connection requests arrive at each edge node according to a Poisson process with rate λ.
- The destination of each connection request is uniformly distributed among all edge nodes except the source.
- The holding time of each connection is exponentially distributed with mean $(1/\mu)$.
- The bandwidth demand of each connection request is one timeslot.
- The number of timeslots in a frame is identical on each fiber link and is equal to L.
- A timeslot is randomly selected from a set of free timeslots on the selected path to be allocated to a connection. Note that a free timeslot on a path is defined as one that is not used on all the links of the path.

In addition, we also assume that the number of overlaid cores in the network is two (i.e., M=2). In this case, there are only two fixed alternate paths between a pair of edge nodes, each passing two hops. We refer the two paths as the first path and the second path, respectively. Note that all these assumptions are also used in the simulation model.

As indicated in Section 2.1, there is no TSI in each core node. This means that the link load or the use of individual timeslots on each link is correlated with each other. To ensure accuracy, it is extremely important to capture this correlation in the analytical model. At the same time, this should not make the model too complicated to be tractable. For this purpose, we use the correlation model presented in [5] and adapt it to the network scenario considered in this work. In the adapted model, a timeslot is analogous to a wavelength in the original model in [5]. It is assumed that the link loads in the network have Markovian spatial correlation, i.e., the timeslots used on a particular link of a path depend only on those used on the previous link of the path. This assumption may capture the correlation effects to a large degree while keeping the model reasonably tractable. It is possible to further extend the correlation effects but at the expense of a more complicated model. Due to the limitation of space, we only give the results that are directly related to our analytical model and refer the readers to [5] for other details on the correlation model.

The notations used in the correlation model are defined as follows:

- $Q(w_f)$: the probability that w_f timeslots are free on a link;
- $S(y_f|x_{pf})$: the conditional probability that y_f timeslots are free on a link of a path, given timeslots are free on the previous link of the path;
- $U(z_c|y_f, x_{pf})$: the conditional probability that z_c connections (timeslots) continue to the current link from the previous link of a path, given x_{pf} timeslots are free on the previous link and y_f timeslots are free on the current link;
- $R(n_f|x_{ff}, y_f, z_c)$: the conditional probability that n_f timeslots are free on a two-hop path, given x_{ff} timeslots are free on the first hop, y_f timeslots are free on the second hop, and z_c connections continue from the first hop to the second hop;
- $T^{(l)}(n_f, y_f)$: the probability that n_f timeslots are free on an l-hop path and y_f timeslots are free on hop l;

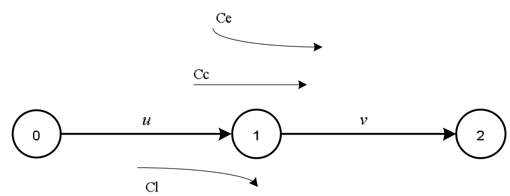

Figure 3 Call arriving and leaving on a two-hop path

Consider a two-hop path consisting of link u (the first link) and link v (the second link), as shown in Figure 3. Let C_l be the number of calls that enter link u at node 0 but leave link u at node 1, C_c be the number of calls that enter link u at node 0 and continue on to link v at node 1, and C_e be the number of calls that enter link v at node 1. Accordingly, the number of calls that use the first link is $C_l + C_c$ and the number of the calls that use the second link is $C_c + C_e$. Since the number of calls on a link cannot exceed the total number of available timeslots (L) in a frame, we have $C_l + C_c \le L$ and $C_c + C_e \le L$. Also, let λ_l be the arrival rate of calls that enter link u at node 0 but leave link v at node 1, λ_c be the arrival rate of calls that enter link u at node 0 and continue on to link v *at* node *1*, and λ_e be the arrival rate of calls that enter link v at node 1. The corresponding Erlang load is denoted by $\rho_l = \lambda_l / \mu$, $\rho_c = \lambda_c / \mu$, and $\rho_e = \lambda_e / \mu$. Therefore, C_l, C_c, and C_e can be characterized by a three-dimensional Markov chain, with each state denoted by an integer triplet (c_l, c_c, c_e) . The steady-state probability of state (c_l, c_c, c_e) as in [10] is given by

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$$\pi(c_l,c_c,c_e)=\frac{\frac{\rho_l^{c_l}}{c_l!}\frac{\rho_c^{c_c}}{c_c!}\frac{\rho_e^{c_e}}{c_e!}}{\sum_{j=0}^{L}\sum_{i=0}^{L-j}\sum_{k=0}^{L-j}\frac{\rho_l^{i}}{i!}\frac{\rho_c^{j}}{j!}\frac{\rho_e^{k}}{k!}},\quad \begin{array}{l}0\le c_l+c_c\le L\\ 0\le c_c+c_e\le L\end{array}$$

Hence, the conditional probabilities defined earlier can be derived as follows, which is similar to in [5].

$$R(n_f|x_{ff},y_f,z_c)=\frac{\binom{x_{ff}}{n_f}\binom{L-x_{ff}-z_c}{y_f-n_f}}{\binom{L-z_c}{y_f}} \tag{1}$$

for $\min(x_{ff},y_f)\ge n_f\ge \max(0,x_{ff}+y_f+z_c-L)$, and is 0 otherwise.

$$\begin{aligned}&U(z_c|y_f,x_{pf})\\&=P(C_c=z_c|C_c+C_e=L-y_f,C_l+C_c=L-x_{pf})\\&=\frac{\pi(L-x_{pf}-z_c,z_c,L-y_f-z_c)}{\sum_{x_c=0}^{\min(L-x_{pf},L-y_f)}\pi(L-x_{pf}-x_c,x_c,L-y_f-x_c)}\end{aligned} \tag{2}$$

$$\begin{aligned}&S(y_f|x_{pf})\\&=P(C_c+C_e=L-y_f|C_l+C_c=L-x_{pf})\\&=\frac{\sum_{x_c=0}^{\min(L-x_{pf},L-y_f)}\pi(L-x_{pf}-x_c,x_c,L-y_f-x_c)}{\sum_{x_c=0}^{L-x_{pf}}\sum_{x_e=0}^{L-x_c}\pi(L-x_{pf}-x_c,x_c,x_e)}\end{aligned} \tag{3}$$

and

$$\begin{aligned}&Q(w_f)=P(C_l+C_c=F-w_f)\\&=\sum_{x_c=0}^{F-w_f}\sum_{x_n=0}^{F-x_c}\pi(F-w_f-x_c,x_c,x_n)\end{aligned} \tag{4}$$

The steady-state probability that n_f timeslots are free on an l-hop path and y_f timeslots are free on hop l can be recursively calculated as

$$\begin{aligned}T^{(l)}(n_f,y_f)=&\sum_{x_{pf}=0}^{L}\sum_{x_{ff}=0}^{L}\sum_{z_c=0}^{L'}R(n_f|x_{ff},y_f,z_c)U(z_c|y_f,x_{pf})\\&\times S(y_f|x_{pf})T^{(l-1)}(x_{ff},x_{pf})\\&L'=\min(L-x_{pf},L-y_f)\end{aligned} \tag{5}$$

Note that the starting point of the above recursion is $T^{(1)}(n_f,y_f)$, which is given by

$$T^{(1)}(n_f,y_f)=\begin{cases}0 & n_f\ne y_f\\ Q(n_f) & n_f=y_f\end{cases}$$

Therefore, the probability that n_f timeslots are free on an l-hop path p is given by

$$Q^{(l)}(n_f)=Q_p(n_f)=\sum_{y_f=0}^{L}T^{(l)}(n_f,y_f) \tag{6}$$

In the correlation model, a path selected for a connection does not depend on the link state on the path. For the fixed shortest-path routing, it is possible to assume that the effect of the blocking probability on the carried traffic load is negligible and the arrival rate on each link is the same in order to make the analysis simple. However, these assumptions become invalid for the least-congested-path routing. In this case, path selection is based on the current network state. The arrival rate on each link is dynamically changing. As a result, there exists no steady state in a strict sense [6]. To address this problem, we use a method based on the Erlang fixed-point method for alternate routing [11] similar to [6]. To describe this method, the following notations need to be defined.

- $R_u^{(1)}/R_v^{(1)}$: the set of all first-paths that pass through link u /link v;
- $R_u^{(2)}/R_v^{(2)}$: the set of all second-paths that pass through link u /link v;
- $R_{u,v}^{(1)}$: the set of all first-paths that pass through link u and link v ;
- $R_{u,v}^{(2)}$: the set of all second-paths that pass through link u and link v ;
- $P_1(p_{ij}^1)$: the probability that a connection between node i and node j is established on the first path, p_{ij}^1;
- $P_2(p_{ij}^2)$: the probability that a connection between node i and node j is established on the second path, p_{ij}^2.

Using the least-congested-path routing, a connection is established on the path with more free timeslots. Hence, if the first path has more free timeslots than the second path, it is selected for the connection. Otherwise, the second path is selected if there is at least one free timeslot on the path. Therefore, we have

$$P_1(p_{ij}^1)=\sum_{\alpha=1}^{L}Q_{p_{ij}^1}(\alpha)\sum_{\beta=0}^{\alpha}Q_{p_{ij}^2}(\beta) \tag{7}$$

$$P_2(p_{ij}^2)=\sum_{\alpha=1}^{L}Q_{p_{ij}^2}(\alpha)\sum_{\beta=0}^{\alpha-1}Q_{p_{ij}^1}(\beta) \tag{8}$$

The arrival rate of calls that enter link u and continue to link v is given by

$$\rho_c(u,v)=\begin{cases}\lambda P_1(p_{ij}^1) & \text{if } u=e_{i1},\ v=e_{1j}\\ \lambda P_2(p_{ij}^2) & \text{if } u=e_{i2},\ v=e_{2j}\\ 0 & \text{otherwise}\end{cases} \tag{9}$$

The arrival rate of the calls that leave link u, which include the calls that use link u in the first or second path but do not continue to link v, is given by

$$\rho_l(u,v) = \begin{cases} \sum_{p_{ij}^1 \in R_u^1} \lambda P_1(p_{ij}^1) - \rho_c(u,v) & \text{if } u = e_{i1} \\ \sum_{p_{ij}^2 \in R_u^2} \lambda P_2(p_{ij}^2) - \rho_c(u,v) & \text{if } u = e_{i2} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

The arrival rate of the calls that enter link *v*, which include the calls that use link *v* in the first or second path but do not continue from link *u* to link *v*, is given by

$$\rho_e(u,v) = \begin{cases} \sum_{p_{ij}^1 \in R_v^1} \lambda P_1(p_{ij}^1) - \rho_c(u,v) & \text{if } v = e_{1j} \\ \sum_{p_{ij}^2 \in R_v^2} \lambda P_2(p_{ij}^2) - \rho_c(u,v) & \text{if } v = e_{2j} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

Given the arrival rates to each link, the blocking probability between edge node *i* and edge node *j* can be calculated as follows.

$$P_b(i,j) = Q_{p_{ij}^1}(0) \times Q_{p_{ij}^2}(0) \quad (12)$$

An iterative algorithm is developed to compute the blocking probability on each path. In the algorithm, a small positive number ε is set as a convergence criterion. The main steps of the algorithm can be described as follows.

1) For each pair of source and destination nodes, initialize $P_b'(i,j) = 0$, $i,j = 1,2,\cdots,N$. For all links, initialize $\rho_l(u,v)$, $\rho_c(u,v)$, and $\rho_e(u,v)$ arbitrarily, $u,v \in E$;
2) Calculate $Q_p(n_f)$ for each path between each pair of source and destination nodes using equations (5) and (6);
3) Calculate the blocking probability $P_b(i,j)$ for each pair of source and destination nodes using equation (12). If $\max\left|P_b(i,j) - P_b'(i,j)\right| < \varepsilon$, terminate the computation. Otherwise, let $P_b'(i,j) = P_b(i,j)$ and go to the next step.
4) Calculate $\rho_l(u,v)$, $\rho_c(u,v)$, and $\rho_e(u,v)$ for each link using equations (9), (10), and (11), and then go back to step 2).

IV. NUMERICAL RESULTS

We verify the accuracy of the proposed analytical model by comparing simulation results with analytical results. Without loss of generality, we consider a 4x4 network with eight timeslots (L=8) in each frame.

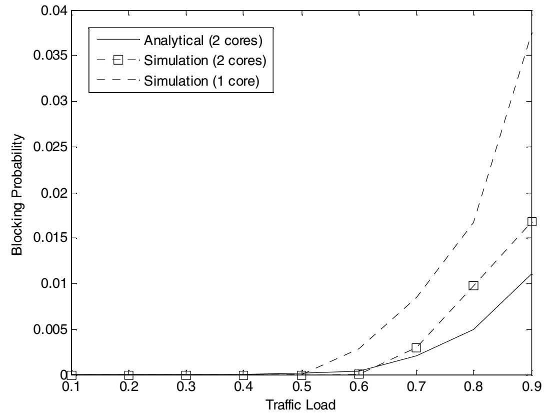

Figure 3 Blocking probability versus traffic load

Figure 3 compares the analytical results and the simulations results in terms of call blocking probability in the network. In computing the analytical results, the convergence factor ε is set to 10^{-10} to ensure accuracy and multiple iterations were performed. In the simulation, each point was obtained with a simulation time of 10^6 timeslots. It is observed that the analytical results are close to the simulation results. Figure 3 also shows the call blocking probabilities with a 4x4 two-core network and a 4x4 one-core network. It is observed that under the same traffic load the two-core network significantly reduces the call blocking probability as compared with the one-care network.

V. CONCLUSIONS

In this paper, we proposed an analytical model to compute the call blocking probability in a class of all-optical overlaid-star TDM networks using a least-congested-path routing strategy. To ensure accuracy, link-load correlation is taken into account in the model. The numerical results show that the proposed model can accurately estimate the call blocking probability in the network. Under the same traffic load, a two-core overlaid network can significantly reduce call blocking probability as compared with a one-core network.

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