

# Resource Criticality Analysis of Static Resource Allocations and Its Applications in WDM Network Planning

James Yiming Zhang, Jing Wu, Gregor v. Bochmann, and Michel Savoie

**Abstract**—Various static resource allocation algorithms have been used in WDM networks to allocate resources such as wavelength channels, transmitters, receivers, and wavelength converters to a given set of static lightpath demands. However, although optimized resource allocations can be obtained, it remains an open issue how to determine which resources are the bottlenecks in achieving better performance. Existing static resource allocation algorithms do not explicitly measure the impact of changes of network resources or lightpath demands on the design objective. We propose such a measurement based on the Lagrangian relaxation framework. We use the optimized values of Lagrange multipliers as a direct measurement of the criticality of resources. Such a quantitative measurement can be naturally acquired along with the optimization process to obtain the optimal solution (or a near-optimal solution) to the static routing and wavelength assignment problem. We investigate three practical applications of the resource criticality (RC) analysis in WDM network planning. In the first application, we use our proposed measurement to identify critical resources and thus to decide the best way to add or reallocate resources. In the second application, we estimate the impact of the addition or removal of lightpath demands on the design objective. This kind of estimation helps to set a proper price for lightpath demands. In the third application, the results of the RC analysis are used to speed up the convergence of the optimization process for different network scenarios.

**Index Terms**—Networks; Assignment and routing algorithms; Combinatorial network design; Network optimization.

## I. INTRODUCTION

**R**OUTING and wavelength assignment (RWA) algorithms, which are used to solve resource allocation problems in WDM networks, have two flavors: static (also called offline) and dynamic (also called online) [1,2]. Static RWA algorithms use a given set of lightpath demands and aim at providing a long-term plan for future traffic [3–11]. In dynamic RWA algorithms, requests to establish or terminate lightpaths arrive dynamically [12,13]. Previous studies have shown the value of using optimized paths that are computed offline to guide online path setups [14]. More specifically, the static RWA solutions form the basis for suboptimal solutions to the dynamic RWA problems [15]. The benefit of dynamic operation of WDM networks in terms of wavelength utilization is significant only at low to moderate traffic loads in sparsely connected networks [16], while static RWA algorithms are important for current and near-future WDM networks, especially when there is no wavelength conversion, the network is highly connected, or the traffic load is moderate to high [16].

Existing RWA algorithms loosely or implicitly use the concept of *resource criticality* (RC) without clearly defining or quantitatively measuring it. The most important strategy of almost all RWA heuristics is to avoid using critical resources and to reserve them for future or other lightpath demands [17–21]. However, there is no clear definition of what RC is and how to calculate it. The difficulty comes from the complicated nature of interactions among competing demands and relations between different resources. Kodialam and Lakshman proposed a minimum interference routing algorithm for MPLS traffic engineering [22]. They consider that when some traffic demands are routed over a given link, the available flow values for one or

Manuscript received March 11, 2009; revised July 15, 2009; accepted July 21, 2009; published August 20, 2009 (Doc. ID 108623).

J. Y. Zhang (e-mail: yizhang@site.uottawa.ca) and G. v. Bochmann (e-mail: bochmann@site.uottawa.ca) are with the School of Information Technology and Engineering, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5.

J. Wu (e-mail: jing.wu@crc.gc.ca) and M. Savoie (e-mail: michel.savoie@crc.gc.ca) are with the Communications Research Centre Canada, Ottawa, Ontario, Canada K2H 852.

Digital Object Identifier 10.1364/JOCN.1.000294

more source–destination node pairs decrease. When these values reach a bottom limit, the link is considered a critical link. However, to simplify the computation, the authors consider only one node pair at a time. They applied the same concept to optical networks in [23]. Ho and Mouftah proposed asynchronous criticality avoidance routing to reduce the mutual interference between lightpaths launched by different node pairs [24]. Under the fixed alternative routing architecture, they measure the criticality of a link by the number of free wavelength channels on that link. When this value drops below a predefined threshold, the corresponding link is treated as a critical resource, and other lightpath demands should then avoid it. However, such a determination of RC is very rough and does not explicitly reflect the impact of a given resource on the design objective. The setting of the threshold to determine a critical resource is arbitrary. Palmieri *et al.* applied the minimum cut algorithm to a graph to identify critical links [25]. Their method measures the criticality of links by the weight for a feasible cut in the graph. However, their method cannot measure the criticality of other resources such as wavelength converters, transmitters, or receivers. Kim *et al.* used a variation of the least congested path in a fixed alternative routing [26]. They used the number of available channels to measure the resource criticality. Mosharaf *et al.* measured the resource criticality as the available wavelength channels on a given link for different service classes and dynamically adjusted their partition [27].

In this paper, we propose a framework with the direct RC definition in static resource allocations in WDM networks, which, at the same time, provides a unified RC measurement for all network resources. Our method has more persuasive mathematical explanations than the previously proposed heuristics, because within the Lagrangian relaxation framework, the optimized values of Lagrange multipliers reflect the RC ([3,28–30] and Section 4.4 on sensitivity in [31]). A method to compute these values for a given resource is also presented. To the best of our knowledge for what is the first time, we provide a quantitative measure of RC and accurately predict the changes to the objective function for what-if situations for RWA problems. We demonstrate three practical applications of the proposed method. In the first application, the optimized Lagrange multipliers are used to identify the critical resources in static RWA schemes and thus to plan the network upgrading or the resources reallocation to better achieve the design objective. Subsequently, results of the RC analysis are used to estimate the impact on the design objective when lightpath demands are added or removed. Thus, a proper price of a lightpath demand can be estimated in WDM network planning. In the last application, re-

sults of the RC analysis are used to speed up the convergence of the optimization process for the similar network scenarios that are in the neighborhood of the original static RWA problem.

This paper is organized as follows: In Section II, we outline an integer linear programming formulation of the static RWA problem; then in Section III, we explain how Lagrange multipliers can be used as an RC measurement, followed by a computation method for the optimized Lagrange multipliers presented in Section IV; in Section V, we show how the optimized Lagrange multipliers are used to identify critical resources; in Section VI, we use results of the RC analysis to estimate the price of a lightpath demand; in Section VII, we use the results of RC analysis to speed up the convergence of the optimization process for similar network scenarios; and we conclude this paper in Section VIII.

## II. INTEGER LINEAR PROGRAMMING FORMULATION OF THE STATIC RWA PROBLEM

We use a mesh network topology with a varying number of wavelength converters (possibly zero) at different nodes. Wavelength converters are installed at a node in a share-per-node manner, which means that any input or output port may use a wavelength converter if one is available. Our network model consists of  $N$  nodes interconnected by  $E$  fibers. Each fiber has  $W$  noninterfering wavelength channels. The fiber between nodes  $i$  and  $j$  is denoted  $e_{ij}$ . The  $c$ th wavelength channel on  $e_{ij}$  is denoted  $w_{ijc}$  ( $0 < c \leq W$ ). The set  $\mathcal{E}$  represents all links in the network. Each link has a pair of fibers, one for each direction. The set  $\mathcal{V}$  represents all the nodes in the network. All nodes are able to perform nonblocking lightpath switching. That means if two wavelength channels using the same wavelength are available in incoming and outgoing fibers at a node, then the node can connect the two wavelength channels regardless of the usage of other wavelength channels at the node. A lightpath can be established between a source and destination node; such a lightpath is defined as a sequence of concatenated wavelength channels. In this paper, we consider only lightpath connections. Traffic grooming below lightpath granularity is not considered. The wavelength channels used by a lightpath on different links are allowed to use different wavelengths, if a wavelength converter is available at the intermediate node. Our model allows more than one lightpath to be set up between a given node pair. The symbol  $s_{sdn}$  denotes the  $n$ th lightpath demand between node pair  $(s, d)$ . The set  $\mathcal{L}$  represents all lightpath demands in the network.

We adopt a penalty-based objective function as in [32], wherein the rejection of demands and the use of

network resources are penalized. Since a certain amount of potential revenue is lost when a request is rejected, the rejection penalty equals the amount of its potential revenue. On the other hand, when a request is accepted, its resource consumption is added as a penalty in the objective function. The resource consumption penalty is the cost of resources used by the lightpath provisioned for the demand.

Our design objective is to minimize the function  $J$ , i.e.,  $\min_{A,\Delta,\Phi}(J)$ , where

$$J = \sum_{s_{sdn} \in \mathcal{L}} [(1 - \alpha_{sdn})P_{sdn} + \alpha_{sdn}C_{sdn}]. \quad (1)$$

For each demand  $s_{sdn}$ , either the penalty of rejecting it ( $P_{sdn}$ ) or the penalty of using resources ( $C_{sdn}$ ) to set up a lightpath is added to the objective function ( $J$ ), depending on  $s_{sdn}$ 's admission status  $\alpha_{sdn}$ . The value of  $\alpha_{sdn}$  is zero if  $s_{sdn}$  is rejected, and  $\alpha_{sdn}$  is one if  $s_{sdn}$  is admitted.

In addition to the design variables  $\alpha_{sdn}$  ( $\forall s_{sdn} \in \mathcal{L}$ ), we introduce the design variables  $\delta_{ijc}^{sdn}$  ( $\forall s_{sdn} \in \mathcal{L}, \forall e_{ij} \in \mathcal{E}, 0 < c \leq W$ ), representing the use of  $w_{ijc}$  by  $s_{sdn}$ , and the design variables  $\phi_i^{sdn}$  ( $\forall s_{sdn} \in \mathcal{L}, \forall i \in \mathcal{V}$ ), representing the use of a wavelength converter at node  $i$  by  $s_{sdn}$ . If  $w_{ijc}$  is used by  $s_{sdn}$ ,  $\delta_{ijc}^{sdn}$  equals one; otherwise,  $\delta_{ijc}^{sdn}$  equals zero. If a wavelength converter is used by  $s_{sdn}$ ,  $\phi_i^{sdn}$  equals one; otherwise,  $\phi_i^{sdn}$  equals zero. We use vector  $A$  to denote the acceptance status of all demands, vector  $\Delta$  to denote their wavelength assignment, and  $F$  to denote their use of wavelength converters. We use  $V$  to denote the design variables ( $A, \Delta, F$ ). For an individual lightpath demand  $s_{sdn}$ , we use  $\Delta_{sdn}$  to denote its wavelength assignment and  $F_{sdn}$  to denote its use of wavelength converters. Now we may define the cost of resources  $C_{sdn}$  as the cost of using wavelength channels and converters:

$$C_{sdn} = \sum_{e_{ij} \in \mathcal{E}} \sum_{0 < c \leq W} d_{ij} \delta_{ijc}^{sdn} + \sum_{i \in \mathcal{V}} o_i \phi_i^{sdn} \quad \forall s_{sdn} \in \mathcal{L}, \quad (2)$$

where  $d_{ij}$  is the cost of using  $w_{ijc}$  and  $o_i$  is the cost of using a wavelength converter at node  $i$ .

The above static RWA problem must conform to the following constraints:

a) Lightpath continuity constraints: If a demand is admitted, the lightpath assigned to it has to be continuous along a path between the source–destination pair. Since the assigned lightpath terminates at the two end nodes, we have

$$\sum_{j \in \mathcal{V}} \sum_{0 < c \leq W} \delta_{ijc}^{sdn} - \sum_{j \in \mathcal{V}} \sum_{0 < c \leq W} \delta_{jic}^{sdn} = \begin{cases} \alpha_{sdn} & \text{if } i = s \\ -\alpha_{sdn} & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} \quad \forall s_{sdn} \in \mathcal{L}. \quad (3)$$

b) Wavelength channel exclusive usage constraints:

$$\sum_{s_{sdn} \in \mathcal{L}} \delta_{ijc}^{sdn} \leq 1 \quad \forall e_{ij} \in \mathcal{E}, \quad 0 < c \leq W. \quad (4)$$

These constraints mean that each wavelength channel can only be used by one lightpath.

c) Transmitter, receiver, and wavelength converter capacity constraints:

$$\sum_{d \in \mathcal{V}} \sum_{0 < n \leq N_{sd}} \alpha_{sdn} \leq T_s \quad \forall s \in \mathcal{V}, \quad (5)$$

$$\sum_{s \in \mathcal{V}} \sum_{0 < n \leq N_{sd}} \alpha_{sdn} \leq R_d \quad \forall d \in \mathcal{V}, \quad (6)$$

$$\sum_{s_{sdn} \in \mathcal{L}} \phi_i^{sdn} \leq F_i \quad \forall i \in \mathcal{V}. \quad (7)$$

The number of lightpaths originating from or terminating at a node must be no more than the number of transmitters or receivers at the node. We assume that all transmitters and receivers operate at any wavelength. The number of transmitters at source node  $s$  is denoted  $T_s$ . The number of receivers at destination node  $d$  is denoted  $R_d$ . The symbol  $N_{sd}$  is the number of lightpath demands between  $(s, d)$ . The number of used converters at a node must be no more than the number of installed converters at the node. The number of wavelength converters at node  $i$  is denoted  $F_i$ .

d) Wavelength conversion constraints:

$$\phi_j^{sdn} = \begin{cases} 1 & \text{if } \exists m, k \in \mathcal{V} \text{ and } b \neq a, \delta_{mja}^{sdn} = \delta_{jkb}^{sdn} = 1 \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in \mathcal{V}. \quad (8)$$

A wavelength converter at an intermediate node  $j$  is used only when different wavelengths are assigned to  $s_{sdn}$  for the incoming and outgoing signals at this node.

### III. LAGRANGE MULTIPLIERS AS A DIRECT RC MEASUREMENT

We use the Lagrange relaxation framework to derive a dual problem (DP) from the primal problem  $\min_{\mathcal{V}}(J)$ , by relaxing the constraints that represent resource limitations. Lagrange multipliers  $\xi_{ijc}$ ,  $\pi_s$ ,  $\theta_d$ , and  $\lambda_i$  are introduced in association with the wave-

length channel exclusive usage constraints (4) and transmitter, receiver, and wavelength converter capacity constraints in (5)–(7). The vectors of Lagrange multipliers  $(\xi_{ijc})$ ,  $(\pi_s)$ ,  $(\theta_d)$ , and  $(\lambda_i)$  are denoted  $\xi$ ,  $\pi$ ,  $\theta$ , and  $\lambda$ , respectively. We use  $M$  to denote all the Lagrange multipliers  $(\xi, \pi, \theta, \lambda)$ . The Lagrangian function  $L$  is defined as [33]

$$\begin{aligned} L(V, M) = & J(V) + \sum_{e_{ij} \in \mathcal{E}} \sum_{0 < c \leq W} \xi_{ijc} \left( \sum_{s_{sdn} \in \mathcal{L}} \delta_{ijc}^{sdn} - 1 \right) \\ & + \sum_{s \in \mathcal{V}} \pi_s \left( \sum_{d \in \mathcal{V}} \sum_{0 < n \leq N_{sd}} \alpha_{sdn} - T_s \right) \\ & + \sum_{d \in \mathcal{V}} \theta_d \left( \sum_{s \in \mathcal{V}} \sum_{0 < n \leq N_{sd}} \alpha_{sdn} - R_d \right) \\ & + \sum_{i \in \mathcal{V}} \lambda_i \left( \sum_{s_{sdn} \in \mathcal{L}} \phi_i^{sdn} - F_i \right). \end{aligned} \quad (9)$$

To analyze the criticality of a given resource, the resource must be modeled in the formulation as a constraint, and then the constraint must be relaxed and be associated with a Lagrange multiplier in the above Lagrangian function. The formulation of the resource constraint should be in the form of the total usage of a resource not exceeding the total number of deployed resources. For example, the total usage of transmitters, receivers, or wavelength converters must not exceed the total number of such deployed resources as in constraints (5)–(7). This is due to the assumption of the shared-per-node structure of the deployment of such resources. The total usage of a given wavelength channel must not exceed the value 1 as in constraints (4).

Define the dual function  $q(M)$  as the infimum of  $L(V, M)$ :

$$q(M) = \min_V [L(V, M)]. \quad (10)$$

The Lagrangian DP is  $\max_{M \geq 0} [q(M)]$ , subject to constraints (3) and (8). We use  $q^*$  to denote the Lagrangian DP's optimal value. The corresponding optimal Lagrange multiplier values are denoted  $M^* = (\xi^*, \pi^*, \theta^*, \lambda^*)$ . The optimal value of the Lagrangian DP is a lower bound of the primal problem [33]:

$$q^*(M^*) = \min_V [L(V, M^*)] \leq \min_V [J(V)]. \quad (11)$$

Since the optimized values of the Lagrange multipliers represent the sensitivity of the design objective  $J$  with respect to the level of a given resource, we can thus use these values as a direct measurement of the criticality of resources. For simplicity, we use the terminology “optimized Lagrange multipliers” to refer to the near-optimal values of Lagrange multipliers. For an optimized Lagrange multiplier  $M_R^*$  corresponding to a given resource  $R$  with a continuous resource level, we have [29]

$$M_R^* = - \frac{dJ^*}{dR}, \quad (12)$$

where  $J^*$  denotes the optimal value of the design objective in the primal problem.

For a resource with discrete levels, the corresponding optimized Lagrange multiplier is only an estimate of the sensitivity of the design objective  $J$  with respect to the level of the resource:

$$M_R^* \cong - \frac{\Delta J^*}{\Delta R}. \quad (13)$$

The optimized Lagrangian multipliers should be considered soft prices that represent the RC and that reflect the interactions between lightpath demands and their competition for given resources. The soft prices are different from resource costs, such as the cost of using wavelength channel  $w_{ijc}$  (denoted  $d_{ij}$ ) and a wavelength converter at node  $i$  (denoted  $o_i$ ) in Eq. (2), and these resource costs do not represent the RC. On the other hand, the soft prices evaluate the RC by using the same measure as resource costs, so the RC and the resource costs are thus unified under the Lagrangian framework.

When the optimized Lagrange multiplier  $M_R^*$  for a given resource  $R$  is known, the impact from any change of the resource on the design objective can be estimated. For example, when a small amount ( $\Delta R$ ) of a critical resource  $R$  is added to the network, the improvement of the design objective can be estimated as  $M_R^* \times \Delta R$ . It should be noted that such an estimation is very rough and applies only to minor resource changes. In general, because interaction among competing demands and relations between different resources are very complicated, the improvement of the design objective needs to be recomputed by solving a new optimization problem.

#### IV. COMPUTATION OF OPTIMIZED LAGRANGE MULTIPLIERS

To obtain optimized Lagrange multipliers, we need to solve the Lagrangian DP. The key to solving the Lagrangian DP is to derive independent subproblems, where the optimal solutions to the subproblems can be computed. By using the fact that  $\delta_{ijc}^{sdn} = \alpha_{sdn} \delta_{ijc}^{sdn}$ ,  $\phi_i^{sdn} = \alpha_{sdn} \phi_i^{sdn}$ , and removing the terms that are independent of the decision variables, the Lagrangian DP becomes (the readers can refer to [32,34] for the mathematical details)



$$q(M) = \min_V \left\{ \sum_{s_{sdn} \in \mathcal{L}} \left[ (1 - \alpha_{sdn}) P_{sdn} + \alpha_{sdn} \left( \sum_{e_{ij} \in \mathcal{E}} \sum_{0 < c \leq W} \delta_{ijc}^{sdn} (\xi_{ijc} + d_{ij}) \right) + \sum_{i \in \mathcal{V}} \phi_i^{sdn} (\lambda_i + o_i) + \pi_s + \theta_d \right] \right\}. \quad (14)$$

The resource allocation to each lightpath becomes independent because resource usage constraints (4)–(7) are relaxed. The complex competition among lightpaths for shared resources does not need to be considered when we allocate resources to each lightpath. Then each subproblem corresponds to the decision of acceptance or rejection of a single lightpath demand, and the associated RWA problem for each accepted lightpath demand. The number of independent subproblems equals the total number of lightpath demands. The optimal value of the relaxed problem is the summation of the optimal values of all lightpath-level subproblems (denoted  $SP_{sdn}$  for the subproblem that corresponds to  $s_{sdn}$ ):

$$q(M) = \sum_{s_{sdn} \in \mathcal{L}} \min_{\alpha_{sdn}} \left[ (1 - \alpha_{sdn}) P_{sdn} + \alpha_{sdn} \min_{\Delta_{sdn}, \Phi_{sdn}} \left( \sum_{e_{ij} \in \mathcal{E}} \sum_{0 < c \leq W} \delta_{ijc}^{sdn} (\xi_{ijc} + d_{ij}) + \sum_{i \in \mathcal{V}} \phi_i^{sdn} (\lambda_i + o_i) + \pi_s + \theta_d \right) \right]. \quad (15)$$

We then use the subgradient method to iteratively maximize the DP and thus to obtain optimized Lagrange multipliers. At the same time, a feasible solution to the original problem (i.e., the primal problem) is derived from the solutions to individual subproblems by using a heuristic algorithm. The overall algorithm is illustrated in Fig. 1. When the computation converges, the optimized Lagrange multipliers are obtained. In addition to the built-in nature of attempting to respect the relaxed constraints in solving the Lagrangian DP, the heuristic algorithm forces the relaxed constraints to be respected. The users can refer to [32–34] for the details.

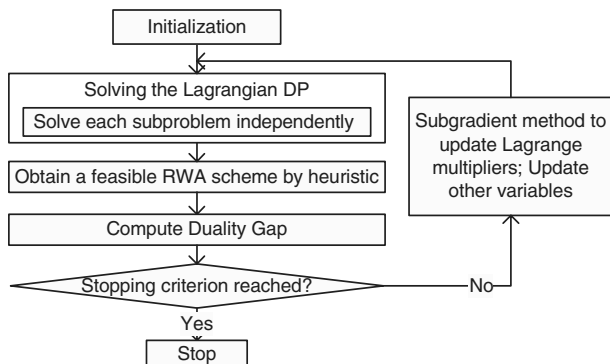


Fig. 1. Schematic depiction of the overall algorithm.

Each subproblem  $SP_{sdn}$  in Eq. (15) can be solved in two steps: lightpath routing and the acceptance versus rejection decision. The first step is to solve the lightpath routing problem:

$$D_{sdn} = \min_{\Delta_{sdn}, \Phi_{sdn}} \left\{ \sum_{e_{ij} \in \mathcal{E}} \sum_{0 < c \leq W} \delta_{ijc}^{sdn} (\xi_{ijc} + d_{ij}) + \sum_{i \in \mathcal{V}} \phi_i^{sdn} (\lambda_i + o_i) \right\}, \quad (16)$$

subject to constraints (3) and (8) for  $s_{sdn}$ . We assign an auxiliary cost  $(\xi_{ijc} + d_{ij})$  to  $w_{ijc}$ . The optimal solution is computed by using the modified minimum cost semi-lightpath algorithm in [34].

The second step is to solve the decision problem:

$$\min_{\alpha_{sdn}} [(1 - \alpha_{sdn}) P_{sdn} + \alpha_{sdn} (D_{sdn} + \pi_s + \theta_d)]. \quad (17)$$

If  $P_{sdn}$  is greater than  $(D_{sdn} + \pi_s + \theta_d)$ , then rejecting  $s_{sdn}$  improves the design objective. In contrast, if  $P_{sdn}$  is smaller, then we accept  $s_{sdn}$  (i.e.,  $\alpha_{sdn} = 1$ ). A tie is broken arbitrarily.

We use the subgradient method to search optimized Lagrange multipliers. The Lagrange multiplier vector  $M = (\xi_{ijc}, \pi_s, \theta_d, \lambda_i)$  is updated toward the direction of its subgradient:

$$M^{(h+1)} = M^{(h)} + \alpha^{(h)} g(M^{(h)}), \quad (18)$$

where  $M^{(h)}$  denotes the value of vector  $M$  obtained at the  $h$ th iteration, and  $\alpha^{(h)}$  denotes the step size in the  $h$ th iteration. The vector  $g(M)$  is the subgradient of the dual function  $q$  with respect to  $M$ , i.e.,  $g(M) = (g_{ijc}(\xi), g_s(\pi), g_d(\theta), g_i(\lambda))$ :

$$g_{ijc}(\xi) = \sum_{s_{sdn} \in \mathcal{L}} \delta_{ijc}^{sdn} - 1, \quad (19)$$

$$g_s(\pi) = \sum_{d \in \mathcal{V}} \sum_{0 < n \leq N_{sd}} \alpha_{sdn} - T_s, \quad (20)$$

$$g_d(\theta) = \sum_{s \in \mathcal{V}} \sum_{0 < n \leq N_{sd}} \alpha_{sdn} - R_d, \quad (21)$$

$$g_i(\lambda) = \sum_{s_{sdn} \in \mathcal{L}} \phi_i^{sdn} - F_i. \quad (22)$$

## V. CRITICAL RESOURCE IDENTIFICATION

The optimized Lagrange multipliers for different resources can be used as a quantitative measurement for the relative significance of the resources. When we add resources with high optimized Lagrange multipliers to the network, the improvement of the design objective is greater than adding resources with low optimized Lagrange multipliers. In this way, the optimized Lagrange multipliers help to identify the bottleneck resources for performance improvements.

Thus, new resources can be more efficiently added to the network, or the existing resources can be reallocated to a more efficient configuration.

To demonstrate this idea, we use optimized Lagrange multipliers to quantify the criticality of the wavelength channels, then add or reallocate wavelength channels to the critical links, and evaluate the improvement on the design objective. The purpose is to demonstrate that, for a given set of static lightpath demands over a given mesh WDM network topology, the optimized Lagrange multipliers can effectively identify the bottleneck resources. Then we verify that the identified resources are actually bottlenecks for performance improvement by performing two simulations: (a) adding resources just to the identified bottlenecks, and (b) reallocating resources from nonbottlenecks to the identified bottlenecks. Both experiments show performance improvements. We use NSFNET, shown in Fig. 2, as an example. The static lightpath demands are shown in Table I, where the horizontal index of the matrix is the source node of a lightpath demand, while the vertical index is the destination node. In the current implementation, the algorithm is terminated when the duality gap does not decrease further after 500 iterations.

We compute optimized Lagrange multipliers for all wavelength channels and then use the average value for the wavelength channels on the same link as a measurement for the link. Conceptually, the wavelength channels (of different wavelength colors) on the same link are all equivalent and thus will have equal impact on the performance (i.e., only the number of wavelength channels on a link matters). We assume that the revenue for each lightpath is 1000 ( $P_{sdn} = 1000$ ), the cost of each wavelength channel is 250 ( $d_{ij} = 250$ ), the cost of each wavelength converter is zero, and more than adequate wavelength converters are available at each node. The number of transmitters and receivers at each node is set to 28 ( $T_i = R_i = 28$ ). Initially, we set the number of wavelengths on each link to 16 ( $W = 16$ ). The average optimized Lagrange multipliers for all links are listed in Table II. The design objective function value that corresponds to this resource allocation scheme is 160,924 with a lower bound being 158,849 as shown by case 1 in Fig. 3.

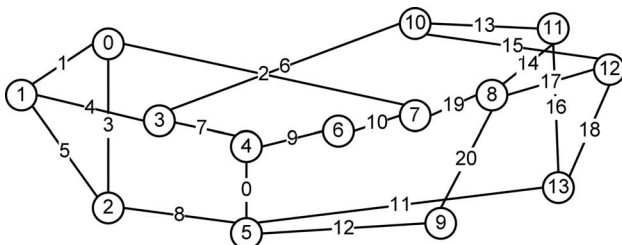


Fig. 2. NSFNET (14 nodes, 21 links).

TABLE I  
LIGHTPATH DEMAND MATRIX

0	1	3	1	3	1	3	0	2	0	3	2	0	3
0	0	0	2	0	2	1	0	1	0	1	0	0	3
3	2	0	3	0	1	2	3	2	3	1	2	2	0
3	1	0	0	1	1	2	3	2	2	1	2	0	3
1	3	0	2	0	1	0	2	0	3	0	1	1	3
1	2	1	3	2	0	1	3	3	1	0	1	0	2
2	2	3	1	3	3	0	0	3	1	2	0	3	3
0	1	2	0	1	0	1	0	0	1	0	0	2	0
3	0	1	3	3	3	1	0	0	2	1	1	1	2
0	0	0	1	2	0	2	0	1	0	1	0	0	3
1	0	0	2	0	3	0	1	0	3	0	3	0	3
2	3	1	1	3	2	3	2	2	2	0	1	3	
2	0	1	0	0	1	2	0	3	0	2	0	0	3
1	1	0	2	1	0	1	3	0	1	2	1	3	0

The design objective values under different resource configurations and/or amounts are compared (see Fig. 3). Note that the top of a bar represents the achieved value of the design objective corresponding to a feasible static RWA scheme obtained by our Lagrangian relaxation and subgradient method; the bottom of a bar represents an optimized lower bound to the design objective. The real global optimal solution of the design objective must be a value lying within the range of the bar. Since the design objective is to minimize the penalty, cases 2 and 4 outperform cases 1 and 3. In

TABLE II  
AVERAGE OPTIMIZED LAGRANGE MULTIPLIERS FOR ALL LINKS

Link No.	Average Optimized Lagrange Multipliers (Direction 1)	Average Optimized Lagrange Multipliers (Direction 2)
<b>0</b>	<b>114.0</b>	<b>120.8</b>
1	0.1	0.2
2	0.4	52.1
3	0.1	0.4
4	0.4	0.4
5	0	0.2
<b>6</b>	<b>179.7</b>	<b>97.1</b>
7	0.4	10.4
8	26.4	0.5
9	0.2	0.2
10	52.2	26.2
<b>11</b>	<b>163.2</b>	<b>143.1</b>
12	14.9	0.5
13	0.6	0.2
14	0.3	0.6
15	0.3	0.4
16	0.3	0.2
17	0.4	0.4
18	0.3	0.2
<b>19</b>	<b>68.6</b>	<b>108.7</b>
20	0.6	0.1

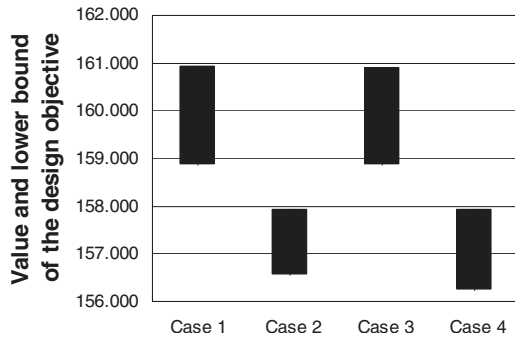


Fig. 3. Achieved design objective values and the lower bounds when additional wavelength channels are added or existing wavelength channels are relocated.

case 2, we add two wavelength channels to the four most critical links identified in Table II, i.e., links 0, 6, 11, and 19. Based on the optimized Lagrange multipliers for these four selected links (shown in Table II), the improvement on the design objective is estimated to be 1990.4 after two channels are added for both directions. The actual improvements after the computation on the bound and the design objective are 2308.0 and 2993.0, respectively. As a comparison, in case 3, we add two wavelength channels to four randomly selected noncritical links, i.e., 1, 5, 9, and 18. It is shown that the design objective is not improved when wavelength channels are added in these noncritical links. In case 4, instead of adding resources, we reallocate two wavelength channels from noncritical links to the four most critical links, i.e., from links 1, 5, 9, and 18 to links 0, 6, 11, and 19. Similar to case 2, the improvement on the design objective is estimated to be 1990.4 based on the optimized Lagrange multipliers. The actual improvements are 2631.0 and 2984.0, respectively. These comparisons show that the optimized Lagrange multipliers not only successfully identify the critical links for the number of wavelength channels, but also provide a good quantitative estimation for the improvement of the design objective.

The reason for adding two wavelength channels (instead of one single wavelength channel) on each of the four selected critical links is that a conclusive improvement of the optimal design objective can be shown only after two wavelength channels are added. The properties of the design problem and our algorithm result in a small duality gap, for example, 1.3% in case 1. The real optimal value of the design objective lies somewhere within the range between the obtained objective value and its lower bound. If the ranges for two cases overlap, although the obtained objective value indicates an improvement, there is a chance that the optimal solution is not improved. In this example, if we add one wavelength channel on each of the four selected critical links, the estimated improvement of the design objective should be 995.2

based on the optimized Lagrange multipliers. This improvement is less than the duality gap of case 1, and a range overlap mentioned above will occur.

In a second example, we use optimized Lagrange multipliers to identify the bottleneck locations of transmitters and receivers. The network topology and lightpath demands are the same as above (see Fig. 2 and Table I). We compute optimized Lagrange multipliers for transmitters and receivers at all nodes, and the results are shown in Table III. Unlike the first example, where the transmitters and receivers are abundant, in the second example, we set the number of transmitters and receivers at all nodes to 20. In this way, transmitters and receivers are critical resources in some nodes. Other parameters remain the same as in the first example. The design objective function value is 166,375 with a lower bound being 166,167 as shown by case 1 in Fig. 4. The optimized Lagrange multipliers help to identify the four bottleneck locations of receivers (nodes 5, 6, 10, and 13) and the four bottleneck locations of transmitters (nodes 4, 6, 8, and 11). We add one more transmitter or receiver at each of their bottleneck locations. The achieved value of the design objective function is improved, shown as case 2 in Fig. 4. Based on the optimized Lagrange multipliers for these four selected transmitters and receivers (shown in Table III), the improvement for the design objective is estimated to be 1718.7. The actual improvements on the bound and the design objective are 2047.0 and 1738.0, respectively. As a comparison, we randomly add the same number of transmitters (for example, at nodes 1, 7, 9, and 10) and receivers (at nodes 0, 2, 11, and 12), and the achieved value of the

TABLE III  
OPTIMIZED LAGRANGE MULTIPLIERS FOR TRANSMITTERS AND RECEIVERS AT ALL NODES

Node No.	Optimized Lagrange Multipliers for Transmitters	Optimized Lagrange Multipliers for Receivers
0	0	0
1	0	0
2	89.1	0
3	57.7	4.0
4	<b>246.1</b>	0
5	41.9	<b>85.0</b>
6	<b>248.7</b>	<b>158.1</b>
7	0	0
8	<b>245.1</b>	0
9	0	0
10	0	<b>249.2</b>
11	<b>247.3</b>	0
12	0	0
13	245.1	<b>239.2</b>

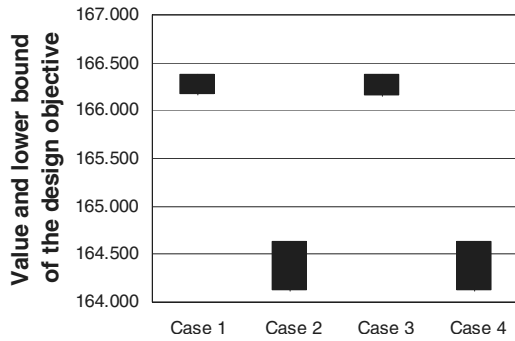


Fig. 4. Achieved design objective values and the lower bounds when additional transmitters and receivers are added or existing transmitters and receivers are relocated.

design objective function is not improved at all, as shown by case 3 in Fig. 4. In the last simulation, we reallocate transmitters from four nonbottleneck locations (nodes 1, 7, 9, and 10) to bottleneck locations (nodes 4, 6, 8, and 11). At the same time, we reallocate receivers from four nonbottleneck locations (nodes 0, 2, 11, and 12) to bottleneck locations (nodes 5, 6, 10, and 13). The improvement on the design objective is the same as adding new transmitters and receivers, as shown by case 4 in Fig. 4. Case 4 indicates that with the help of the optimized Lagrange multipliers, even without adding new resources, the design objective can be improved by reallocating existing resources. Similarly to case 2, the improvement on the design objective is estimated to be 1718.7 based on the optimized Lagrange multipliers. The actual improvements on the bound and the design objective are 2057.0 and 1735.0, respectively. Once again, these comparisons show that the optimized Lagrange multipliers not only successfully identify the critical nodes for the number of transmitters or receivers, but also provide a good quantitative estimation for the improvement of the design objective.

The reason for choosing four critical nodes to add transmitters or receivers to can be explained by the property of point-to-point lightpath demands. Adding a transmitter at one source node and a receiver at one destination node only potentially affects the lightpath demands originated at the source node and terminated at the destination node. No other lightpath demand is affected. For example, if we add one transmitter at node 6 and one receiver at node 10, only two lightpath demands are potentially affected under the traffic pattern in Table I. It is very unlikely to result in improvement. In contrast, when we add transmitters and receivers at a group of nodes, the chance of improvement is much greater. Because the lightpath demands are point to point, the number of added transmitters should be the same as the number of added receivers.

We also investigated an incremental deployment of

additional resources. We add one transmitter and one receiver at each of the identified bottleneck locations as in case 2 in Fig. 5 (the same as in Fig. 4). Then we add one more transmitter and one more receiver at each of the bottleneck locations (shown in case 3 in Fig. 5). Last, we add a third transmitter and a third receiver at each of the bottleneck locations (shown in case 4 in Fig. 5). The original case is shown as case 1 in Fig. 5. In each incremental deployment, the improvement on the design objective is estimated to be 1718.7 based on the optimized Lagrange multipliers. The computed improvement on the design objective is, for the achieved values, 1737.0, 1490.0, and 993.0, respectively; for the lower bound, 1700.0, 1478.0, and 1366.0, respectively. The optimized Lagrange multipliers provide a good quantitative estimation for the improvement on the design objective, although the real improvement becomes smaller and smaller when the number of transmitters and receivers approaches saturation.

In the third example, we use optimized Lagrange multipliers to quantify the criticality of wavelength converters. The same network topology and lightpath demands are used as in the previous two examples. We compute optimized Lagrange multipliers for wavelength converters at all nodes (results are shown in Table IV). Unlike the previous two examples, where the wavelength converters are abundant, in the third example, we set its number at all nodes to 1. The results show that wavelength converters in a static RWA are not critical, and their contribution to the design objective is very minimal, which is consistent with other studies [34,35].

## VI. ESTIMATION OF THE PRICE OF A LIGHTPATH DEMAND

From a network operator's point of view, a proper price of a lightpath demand should consider the accumulative resource costs and most importantly the RC of the resources it uses. The results of the RC analysis represent the interactions between lightpath de-

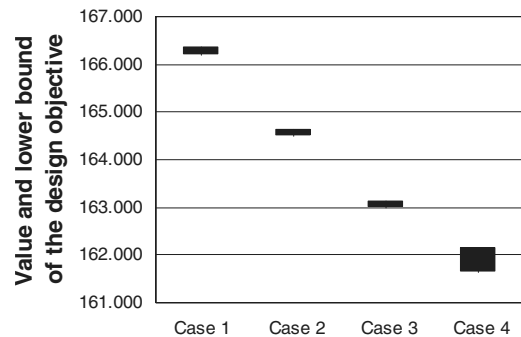


Fig. 5. Achieved design objective values and the lower bounds when additional transmitters and receivers are incrementally deployed.



TABLE IV  
OPTIMIZED LAGRANGE MULTIPLIERS FOR WAVELENGTH CONVERTERS AT ALL NODES

Node No.	Optimized Lagrangian Multipliers	Node No.	Optimized Lagrangian Multipliers
0	0.36	7	0.36
1	0.09	8	0.35
2	0	9	0
3	0	10	0
4	0	11	0
5	0	12	0
6	0	13	0

mands and their competitions for given resources. A network operator wants to charge higher prices for high-demand resources than for low-demand resources. To distinguish the RC cost from the real cost of resources and avoid confusion, we use the term “composite price” to refer to the summation of the cost of a lightpath demand that reflects the RC and the real-world cost. We use results of the RC analysis to estimate the composite price of a lightpath demand. Note that the real-world resource cost of a lightpath is computed as  $C_{sdn}$  in Eq. (2) plus the fixed cost of using a transmitter and a receiver. The resource cost does not contain the RC information, and they are only parameters to compute the composite price. In [36], our resource cost is called “the network link cost”; our composite price is called “the routing link cost” and is measured by the sharing of protection capacity.

The decomposed Lagrangian DP in Eq. (15) provides a method to estimate the composite price of a lightpath demand. We use two slightly different methods to estimate the composite price of a lightpath demand: one for a lightpath demand using a known RWA scheme, and the other for using an unknown RWA scheme.

For a lightpath demand  $s_{sdn}$  using the RWA scheme  $(\Delta_{sdn}, F_{sdn})$ , its composite price is given as  $\sum_{e_{ij} \in \mathcal{E}} \sum_{0 < c \leq W} \delta_{ijc}^{sdn} (\xi_{ijc} + d_{ijc}) + \sum_{i \in \mathcal{V}} \phi_i^{sdn} (\lambda_i + o_i) + \pi_s + \theta_d$ .

For a lightpath demand  $s_{sdn}$  whose RWA scheme is unknown, a solution  $(\Delta_{sdn}, F_{sdn})$  to the optimization problem  $\min_{\Delta_{sdn}, F_{sdn}} [\sum_{e_{ij} \in \mathcal{E}} \sum_{0 < c \leq W} \delta_{ijc}^{sdn} (\xi_{ijc} + d_{ijc}) + \sum_{i \in \mathcal{V}} \phi_i^{sdn} (\lambda_i + o_i)]$  needs to be obtained first. The optimization problem is subject to constraints (3) and (8). Then, we estimate its composite price in the same way as for a lightpath demand with a known RWA scheme.

After the composite price of a lightpath demand is computed, we can determine the effect of adding or removing lightpath demands on the design objective. We use NSFNET shown in Fig. 2 as an example. The static lightpath demands are shown in Table I. We use the following parameters:  $P_{ij0} = 1000$ ,  $F_i = 1$ ,  $W = 16$ ,  $T_i$

$= R_i = 20$ ,  $d_{ij} = 250$ , and an unlimited number of wavelength converters at all nodes. To make a fair comparison, we compare only lightpath demands with one fiber hop. The lowest composite price of the one-hop lightpath demands is 250. It is achieved between node pairs (0, 1), (9, 8), (1, 3), and (10, 11). The highest composite price of the one-hop lightpath demands is 747. It is achieved between node pairs (11, 10), (11, 13), (4, 5), and (13, 5). Figure 6 shows the achieved value and a lower bound of the design objective when additional lightpath demands are added. Case 1 is the achieved value and a lower bound for the original lightpath demands matrix shown in Table I. In case 2, one lightpath demand is added for each of the four node pairs that have the lowest composite price. In case 3, one lightpath demand is added for each of the four node pairs that have the highest composite price. Because the composite prices are lower than the revenue of a lightpath demand, which is set to 1000, the additional lightpath demands are all accepted. Then the achieved value of the design objective reflects the additional resource cost. As we expected, adding lightpath demands with higher composite prices introduces more negative impact than adding lightpath demands with lower composite prices.

## VII. CONVERGENCE SPEEDUP OF THE OPTIMIZATION PROCESS FOR SIMILAR SCENARIOS

An important feature of our proposed method for measuring RC is that for two similar scenarios their RC analysis results are similar, too. A scenario is defined as a set of lightpath demands for a configuration of network resources. When there is a change in the lightpath demands or the network resources, a new scenario is formed. Our method preserves the neighborhood property of RC analysis results, which means that when a new scenario is in the neighborhood of the previous one, our RC analysis produces good estimates for the new scenario.

The neighborhood property of our RC analysis lets Lagrange multipliers be reused to save computation

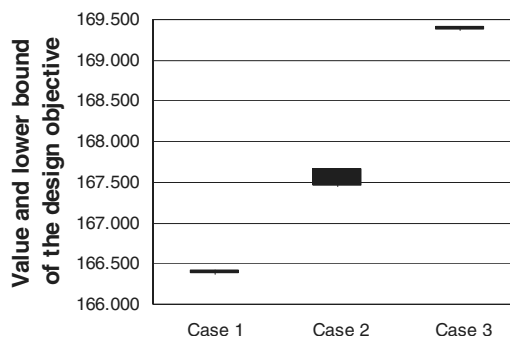


Fig. 6. Achieved value and a lower bound of the design objective when additional lightpath demands are added.

time in scenario studies. To study a new scenario, instead of solving a new optimization process, the Lagrange multipliers obtained from the previous scenario can be reused to speed up the optimization process. Note that most other optimization methods such as the linear programming relaxation method need to re-solve the whole optimization problem again for any changes [37,38]. This feature makes our method of higher practical value in what-if scenario studies.

In the following example, we demonstrate the improvement of the convergence speed by reusing Lagrange multipliers to save the computation time in similar scenarios. We use the Pan-European network with 28 nodes and 61 links (shown in Fig. 7). The parameters used in the example are  $P_{ij0}=1000.0$  for all lightpath demands,  $d_{ij}=10.0$  for all links,  $F_i=1$ ,  $T_i=R_i=18$ ,  $o_i=0$ ,  $t_i=r_i=0$ , for all nodes, and  $W=16$ . We run the heuristic algorithm once every ten iterations to obtain a feasible solution. The lightpath demands for the first and second scenarios are shown in Tables V and VI. The second scenario is a minor variation of the first one. The variations of the lightpath demands from the first scenario to the second one are highlighted by bold and italic numbers in Table VI.

We compare the number of iterations required for the convergence of the optimization process. When studying the second scenario, we use two different strategies in initializing the Lagrange multipliers:

TABLE V  
LIGHTPATH DEMAND MATRIX IN THE PREVIOUS SESSION FOR THE MULTIPLIER INITIALIZATION

0	0	2	0	0	0	2	0	2	0	0	2	0	2	2	0	2	0	0	0	2	0	0	1	0	2	2	2					
0	0	0	2	2	2	0	0	0	2	0	0	0	2	0	2	0	2	2	0	2	2	0	2	2	0	2	2	0				
2	2	0	2	0	2	2	0	2	2	0	2	2	0	2	2	0	2	2	0	2	0	1	1	0	0	0	0	1				
2	0	0	0	0	2	2	2	0	2	2	0	2	2	0	2	2	0	1	0	1	0	0	0	0	0	0	0	0				
0	2	0	2	0	0	2	0	2	0	0	0	2	0	2	0	0	0	2	0	2	0	1	0	0	0	2	0	2				
0	2	0	2	2	0	0	2	2	0	0	0	2	2	2	0	0	2	0	0	2	1	0	0	0	2	2	0	2				
1	1	2	0	2	2	0	2	0	1	0	2	2	2	0	0	1	2	0	0	0	2	0	0	1	0	1	0	1				
2	0	1	2	0	0	0	0	2	1	0	2	0	0	2	0	2	0	2	0	2	2	2	1	2	1	1	1	1				
2	0	0	2	2	2	0	0	1	0	0	0	0	2	0	2	2	1	0	2	0	1	0	0	2	2	0	2	2				
0	0	0	0	1	0	1	0	0	0	0	0	0	2	0	0	1	0	1	0	0	0	0	1	0	0	2	2	2				
0	0	0	1	0	2	0	0	0	2	0	2	0	2	0	1	0	0	0	2	2	1	0	1	0	2	0	0	0				
1	2	0	0	2	1	2	1	1	1	0	0	2	1	0	2	1	2	1	0	0	0	2	0	1	0	2	0	1				
1	0	0	1	0	0	1	0	2	0	1	0	0	2	2	1	1	0	0	0	1	2	2	0	0	2	0	0	2				
0	0	0	1	0	0	0	2	0	0	1	0	2	0	0	1	0	0	0	0	1	2	2	2	0	0	0	0	0				
2	0	0	0	0	1	2	1	1	0	1	0	2	0	0	1	2	0	1	1	0	2	0	2	0	1	0	2	0	1			
0	1	0	2	1	0	0	2	2	0	0	0	1	2	0	0	0	1	0	0	2	1	1	1	1	1	0	0	0				
0	2	0	1	0	0	0	1	0	2	0	0	0	2	0	1	0	0	0	2	0	1	0	0	1	0	0	0	0				
1	1	2	0	2	2	0	0	2	0	1	0	2	2	2	0	0	0	2	0	0	0	0	0	0	0	0	0	2	2			
2	0	1	2	0	0	0	0	2	1	0	2	0	0	2	0	2	0	2	0	2	0	0	0	2	1	0	0	0	2	1		
0	0	0	1	0	2	0	0	0	2	0	2	0	2	0	1	0	0	0	2	1	0	0	2	0	2	0	2	0	2	0		
0	0	0	0	1	0	1	0	0	0	0	0	2	0	0	1	0	1	0	0	0	0	2	0	0	2	0	0	2	1	1		
1	0	0	1	0	0	1	0	2	0	1	0	0	2	2	1	1	1	0	0	0	0	2	0	2	0	0	2	0	0	2		
1	0	2	0	0	2	2	0	2	0	1	0	1	0	2	0	0	1	0	0	0	2	0	2	0	2	0	1	0	1	0	1	
0	0	1	0	0	1	0	2	0	1	0	0	2	0	2	0	2	0	0	0	2	0	0	0	2	0	0	1	0	1	0	1	
0	1	0	2	0	1	2	0	0	1	0	0	1	0	2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	
2	0	0	1	0	0	2	1	1	1	0	2	1	0	1	0	2	0	0	1	2	0	0	1	2	0	0	0	0	0	0	0	0
0	1	2	0	0	2	0	0	2	0	1	0	2	0	0	1	0	0	0	1	0	1	0	1	0	1	0	1	2	0	2	2	0
2	2	0	0	2	1	2	0	0	1	0	2	0	1	0	2	0	0	1	2	0	0	1	2	0	0	1	2	0	2	0	0	0

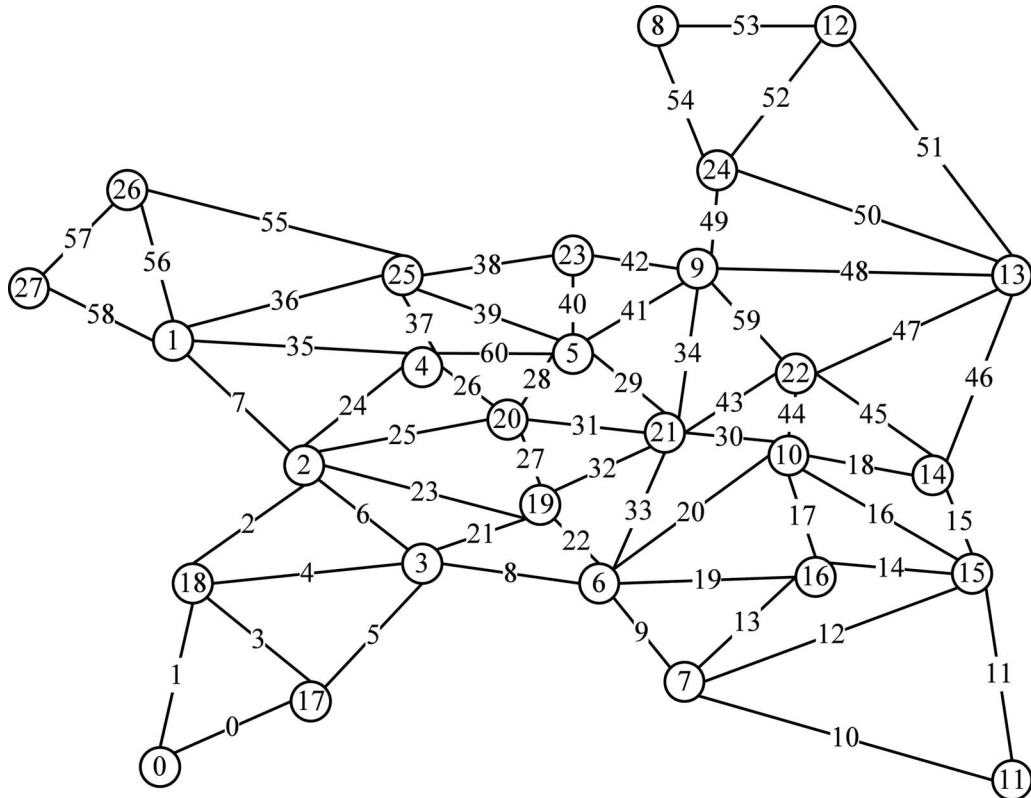


Fig. 7. Pan-European network with 28 nodes and 61 links.

TABLE VI  
CURRENT LIGHTPATH DEMAND MATRIX

0	1	2	0	0	0	2	0	2	0	0	2	0	2	0	0	2	0	0	2	0	0	1	0	2	2	2	
0	0	0	2	2	0	0	2	0	0	0	2	0	2	0	0	2	2	0	2	2	0	2	0	2	0	0	2
2	2	0	2	0	2	2	0	2	2	0	2	2	2	0	2	2	2	0	2	0	2	0	1	1	1	1	
2	0	0	0	2	2	1	2	0	2	2	2	0	2	2	2	0	1	0	1	0	0	0	0	0	0	0	
0	2	0	2	0	0	2	0	2	0	0	2	0	2	0	1	0	0	0	2	0	1	0	0	0	2	2	
0	2	0	0	2	2	1	0	0	2	2	2	0	0	2	1	0	1	0	2	2	1	0	1	0	2	2	
1	1	2	0	2	0	2	0	1	0	2	2	2	0	1	2	0	0	0	2	0	1	0	1	0	1	1	
2	0	1	2	0	0	0	2	1	0	1	0	0	2	0	2	0	2	2	1	2	1	1	1	1	1	1	
2	0	0	2	2	0	0	1	0	0	0	0	2	0	2	2	1	0	2	0	1	0	0	2	2	2	2	
0	0	0	1	0	1	0	0	0	0	0	2	0	0	1	0	1	0	0	0	0	1	0	0	2	2	2	
0	0	0	1	0	1	0	0	2	0	2	0	2	0	1	0	0	0	2	1	0	0	0	2	0	0	2	
1	2	0	2	1	2	1	1	1	0	0	2	1	0	2	1	2	1	0	0	0	2	0	1	0	2	1	
1	0	1	0	0	1	0	2	0	1	0	0	1	2	1	1	0	0	0	1	2	2	0	0	2	0	2	
0	0	0	1	0	2	0	0	2	0	2	0	2	0	1	0	0	0	2	1	0	0	0	2	0	2	0	
0	0	0	1	0	2	0	0	0	0	2	0	2	1	0	1	0	0	0	2	0	0	2	0	2	0	0	
1	0	0	1	0	2	0	1	0	0	2	2	1	1	1	1	1	0	0	0	2	0	2	0	2	0	2	
1	0	2	0	2	2	2	0	2	0	1	0	2	0	0	1	0	0	0	2	0	2	0	1	0	2	0	
0	0	2	0	0	1	0	2	0	1	0	2	2	0	2	0	0	2	0	0	0	2	0	1	0	2	0	
0	1	0	2	0	1	0	0	2	2	0	2	0	1	0	2	0	0	0	2	0	0	1	0	2	0	1	
0	1	0	2	0	1	0	0	2	2	0	2	0	1	0	0	0	0	0	2	0	0	1	0	2	0	1	
2	0	0	1	0	0	2	1	1	1	0	2	2	1	1	1	1	0	0	0	2	0	2	0	2	0	2	
2	0	0	0	1	0	2	0	0	0	2	0	2	1	0	1	0	0	0	2	0	0	2	0	2	0	0	
1	0	0	1	0	2	0	1	0	0	2	2	1	1	1	1	1	0	0	0	2	0	2	0	2	0	2	
1	0	2	0	2	2	2	0	2	0	1	0	2	0	0	1	0	0	0	2	0	2	0	1	0	2	0	
0	0	2	0	0	1	0	2	0	1	0	2	2	0	2	0	0	2	0	0	2	0	1	0	2	0	1	
0	1	0	2	0	1	2	0	0	1	0	2	1	0	0	0	0	0	0	2	0	0	1	0	2	0	0	
2	0	0	1	0	0	2	1	1	1	0	2	2	1	0	2	0	1	0	2	0	0	1	2	0	0	0	
0	1	2	0	0	2	0	0	0	0	2	0	0	1	0	0	1	0	1	0	1	0	1	2	0	2	2	
2	2	0	0	2	1	2	0	0	1	0	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	0	

Case A: initializing all the Lagrange multipliers to zeros.

Case B: initializing the Lagrange multipliers to the obtained optimized Lagrange multipliers from the first scenario.

A dramatic difference in the convergence speed is observed between the two initialization schemes (shown in Fig. 8). In case B, the computation almost reached the optimal values after 40 iterations. In contrast, in case A, the optimization process does not converge to a similar duality gap until after 400 iterations.

Please note that we have observed similar behaviors in many other computation examples, and the readers are referred to [32,34] for more network scenarios.

VIII. CONCLUSIONS

We proposed the use of optimized Lagrange multipliers as a direct measure of resource criticality (RC) in the context of the routing and wavelength assignment (RWA) problem for WDM networks. Since the optimized Lagrange multipliers reflect the impact of resources on the design objective, we showed how the

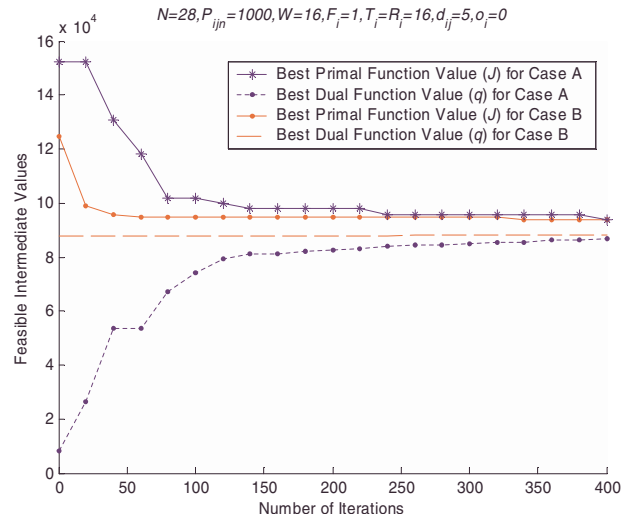


Fig. 8. (Color online) Convergence speed comparison between different multiplier initialization schemes.

optimized Lagrange multipliers obtained in the Lagrangian relaxation and subgradient method can be used to quantify the RC in the static RWA problem. The proposed RC measurement and its computation method can be applied to a wide range of static RWA problems that are formulated as integer linear programming problems. Simulation results indicate that the optimized Lagrange multipliers successfully identify critical resources and thus help to plan network reconfigurations by adding new resources or reallocating existing resources and help to estimate the impact from the change of network traffics. We also demonstrated applications of using the results of the RC analysis to set the proper price of a lightpath demand and to speed up convergence in scenario studies.

REFERENCES

- [1] H. Zang, J. P. Jue, and B. Mukherjee, "Review of routing and wavelength assignment approaches for wavelength-routed optical WDM networks," *Opt. Networks Mag.*, vol. 1, no. 1, pp. 47–60, Jan. 2000.
- [2] R. Dutta and G. N. Rouskas, "Survey of virtual topology design algorithm for wavelength-routed optical networks," *Opt. Networks Mag.*, vol. 1, no. 1, pp. 73–89, Jan. 2000.
- [3] X. Guan, S. Guo, Q. Zhai, W. Gong, and C. Qiao, "A new method for solving routing and wavelength assignment problems in optical networks," *J. Lightwave Technol.*, vol. 25, no. 8, pp. 1895–1909, Aug. 2007.
- [4] T. F. Noronha and C. C. Ribeiro, "Routing and wavelength assignment by partition colouring," *Concr. Library Int.*, vol. 171, no. 3, pp. 797–810, June 2006.
- [5] N. Skorin-Kapov, "Routing and wavelength assignment in optical networks using bin packing based algorithms," *Concr. Library Int.*, vol. 177, no. 2, pp. 1167–1179, March 2007.
- [6] H. Choo, V. V. Shakhov, and B. Mukherjee, "Routing and wavelength assignment in optical WDM networks with maximum quantity of edge disjoint paths," *Photonic Network Commun.*, vol. 12, no. 2, pp. 145–152, Sept. 2006.
- [7] Y. Wang, T. H. Cheng, and M. H. Lim, "A Tabu search algorithm for static routing and wavelength assignment problem," *IEEE Commun. Lett.*, vol. 9, no. 9, pp. 841–843, Sept. 2005.
- [8] L. D. Belgacem and N. Puech, "Solving large size instances of

- the RWA problem using graph partitioning," in *Int. Conf. on Optical Network Design and Modeling 2008. ONDM 2008*, Vilanova i la Geltrú, Spain, March 12–14, 2008, pp. 1–6.
- [9] S. Antonakopoulos and L. Zhang, "Heuristics for fiber installation in optical network optimization," in *IEEE Global Telecommunications Conf., 2000. GLOBECOM '07*, Washington, DC, Nov. 26–30, 2007, pp. 2342–2347.
- [10] M. A. Ezzahdi, S. A. Zahr, M. Koubàa, N. Puech, and M. Gagnaire, "LERP: a quality of transmission dependent heuristic for routing and wavelength assignment in hybrid WDM networks," *15th Int. Conf. Computer Communications and Networks, 2006. ICCCN 2006, Proceedings*, Arlington, VA, Oct. 9–11, 2006, pp. 125–136.
- [11] B. Jaumard, C. Meyer, and B. Thiongane, "Comparison of ILP formulations for the RWA problem," *Opt. Switching Networking*, vol. 4, nos. 3–4, pp. 157–172, Nov. 2007.
- [12] A. Jukan and G. Franzl, "Path selection methods with multiple constraints in service-guaranteed WDM networks," *IEEE/ACM Trans. Netw.*, vol. 12, no. 1, pp. 59–72, Feb. 2004.
- [13] S. H. Ngo, X. Jiang, and S. Horiguchi, "An ant-based approach for dynamic RWA in optical WDM networks," *Photonic Network Commun.*, vol. 11, no. 1, pp. 39–48, Jan. 2006.
- [14] A. Elwalid, D. Mitra, I. Saniee, and I. Widjaja, "Routing and protection in GMPLS networks: from shortest paths to optimized designs," *J. Lightwave Technol.*, vol. 21, no. 11, pp. 2828–2838, Nov. 2003.
- [15] A. E. Ozdaglar and D. P. Bertsekas, "Routing and wavelength assignment in optical networks," *IEEE/ACM Trans. Netw.*, vol. 11, no. 2, pp. 259–272, Apr. 2003.
- [16] A. Zapata-Beghelli and P. Bayvel, "Dynamic versus static wavelength-routed optical networks," *J. Lightwave Technol.*, vol. 26, no. 20, pp. 3403–3415, Oct. 2008.
- [17] X. Chu and B. Li, "Dynamic routing and wavelength assignment in the presence of wavelength conversion for all-optical networks," *IEEE/ACM Trans. Netw.*, vol. 13, no. 3, pp. 704–715, June 2005.
- [18] T. Matsumoto and T. Takenaka, "Overlap degree aware routing in all-optical routing networks," *IEICE Trans. Commun.*, vol. E91-B, no. 1, pp. 212–220, Jan. 2008.
- [19] S. Kim, X. J. Zhang, and S. S. Lumetta, "Rapid and efficient protection for all-optical WDM mesh networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 9, Suppl., pp. 68–82, Dec. 2007.
- [20] M. Gagnaire, M. Koubàa, and N. Puech, "Network dimensioning under scheduled and random lightpath demands in all-optical WDM networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 9, Suppl., pp. 58–67, Dec. 2007.
- [21] G. Shen and W. D. Grover, "Dynamic path-protected service provisioning in optical transport networks with a limited number of add/drop ports and transmitter tenability," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 6, Suppl., pp. 121–134, Aug. 2007.
- [22] M. Kodialam and T. V. Lakshman, "Minimum interference routing with applications to MPLS traffic engineering," in *IEEE INFOCOM 2000, 19th Annu. Joint Conf. of the IEEE Computer and Communications Societies, Proceedings*, Tel Aviv, Israel, March 26–30, 2000, vol. 2, pp. 884–893.
- [23] M. Kodialam and T. V. Lakshman, "Integrated dynamic IP and wavelength routing in IP over WDM networks," in *IEEE INFOCOM 2001, 20th Annu. Joint Conf. of the IEEE Computer and Communications Societies, Proceedings*, Anchorage, AK, Apr. 22–26, 2001, vol. 1, pp. 358–366.
- [24] P. H. Ho and H. T. Mouftah, "A novel distributed control protocol in dynamic wavelength-routed optical networks," *IEEE Commun. Mag.*, vol. 40, no. 11, pp. 38–45, Nov. 2002.
- [25] F. Palmieri, U. Fiore, and S. Ricciardi, "A minimum cut interference-based integrated RWA algorithm for multi-constrained optical transport networks," *J. Network Syst. Manage.*, vol. 16, no. 4, pp. 421–448, Dec. 2008.
- [26] J. S. Kim, D. C. Lee, and H. Sridhar, "Route-metric-based dynamic routing and wavelength assignment for multifiber WDM networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 12, Suppl., pp. 56–68, Dec. 2006.
- [27] K. Mosharaf, J. Talim, and I. Lambadaris, "Optimal resource allocation and fairness control in all-optical WDM networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 8, pp. 1496–1507, Aug. 2005.
- [28] S. S. W. Lee, M. C. Yuang, and P. L. Tien, "A Lagrangean relaxation approach to routing and wavelength assignment for multi-granularity optical WDM networks," in *IEEE Global Telecommunications Conference 2004. GLOBECOM '04*, Dallas, TX, Nov. 29–Dec. 3, 2004, vol. 3, pp. 1936–1942.
- [29] P. B. Luh, D. J. Hoitomt, E. Max, and K. R. Pattipati, "Schedule generation and reconfiguration for parallel machines," *IEEE Trans. Rob. Autom.*, vol. 6, no. 6, pp. 687–696, Dec. 1990.
- [30] A. Lucena, "Lagrangian relax-and-cut algorithms," in *Handbook of Optimization in Telecommunications*, M. G. C. Resende and P. M. Pardalos, eds., New York, NY: Springer Science and Business Media, 2006, pp. 129–146.
- [31] D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, 3rd ed., New York, NY: Springer, 2008.
- [32] J. Y. Zhang, J. Wu, G. Bochmann, and M. Savoie, "Grade-of-service differentiated static resource allocation schemes in WDM networks," *Opt. Switching Networking*, vol. 5, nos. 2/3, pp. 107–122, June 2008.
- [33] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed., Belmont, MA: Athena Scientific, 1999.
- [34] Y. Zhang, O. Yang, and H. Liu, "A Lagrangean relaxation and subgradient framework for the routing and wavelength assignment problem in WDM networks," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 9, pp. 1752–1765, Nov. 2004.
- [35] J. Y. Zhang, J. Wu, G. V. Bochmann, and M. Savoie, "A proof of wavelength conversion not improving the Lagrangian bound of the static RWA problem," *IEEE Commun. Lett.*, vol. 13, no. 5, pp. 345–347, May 2009.
- [36] E. Bouillet, J. F. Labourdette, R. Ramamurthy, and S. Chaudhuri, "Lightpath re-optimization in mesh optical networks," *IEEE/ACM Trans. Netw.*, vol. 13, no. 2, pp. 437–447, Apr. 2005.
- [37] R. M. Krishnaswamy and N. Sivarajan, "Algorithms for routing and wavelength assignment based on solutions of LP-relaxations," *IEEE Commun. Lett.*, vol. 5, no. 10, pp. 435–437, Oct. 2001.
- [38] M. D. Swaminathan and K. N. Sivarajan, "Practical routing and wavelength assignment algorithms for all optical networks with limited wavelength conversion," in *IEEE Int. Conf. on Communications, 2002. ICC 2002*, New York, NY, Apr. 2–May 2002, vol. 5, pp. 2750–2755.



**James Yiming Zhang** obtained the Ph.D. degree in electrical engineering from the University of Ottawa, Ontario, Canada. He received both his B.S. and M.S. degrees in electronic science from Zhejiang University, China. He is currently working as a post-doctoral fellow at the School of Information Technology and Engineering of the University of Ottawa. His research interests include optimization algorithms for communication networks and manufacturing processes, object detection, recognition and tracking of images, machine learning, high-speed digital hardware design, and natural and biometrics.





**Jing Wu** (M'97-SM'08) obtained a B.Sc. degree in information science and technology in 1992 and a Ph.D. degree in systems engineering in 1997, both from Xi'an Jiao Tong University, China. He is now a research scientist at the Communications Research Centre Canada (Ottawa, Canada), an Agency of Industry Canada. In the past, he worked at Beijing University of Posts and Telecommunications (Beijing, China) as a faculty member, Queen's University (Kingston, Canada) as a postdoctoral fellow, and Nortel Networks Corporate (Ottawa, Canada) as a system design engineer. Currently, he is also appointed as an adjunct professor at the University of Ottawa, School of Information Technology and Engineering. He has contributed more than 70 conference and journal papers. He holds three patents on Internet congestion control, and one patent on control plane failure recovery. His research interests include control and management of optical networks, protocols and algorithms in networking, and optical network performance evaluation and optimization. Dr. Wu is a co-chair of the subcommittee on "network architecture, management and applications" of the Asia Communications and Photonics Exhibit and Conference (ACP), which will take place November 2–6, 2009, in Shanghai, China. Dr. Wu is a member of the technical program committees for many international conferences such as IEEE ICC and IEEE GLOBECOM.



**Gregor v. Bochmann** (M'79–F'95) has been a professor at the School of Information Technology and Engineering at the University of Ottawa since January 1998, after 25 years at the University of Montreal. He is a fellow of the IEEE and ACM and a member of the Royal Society of Canada. He did research on programming languages, compiler design, communication protocols, and software engineering and published many papers in these areas. He was also actively involved in the standardization of formal description techniques for communication protocols and services in the 1980s. He had many research projects in collaboration with industry and, from 1989 to 1997, held the Hewlett–Packard–NSERC–CITI Industrial Research Chair on communication protocols at the University of Montreal. His present work is the area of software engineering for distributed systems, peer-to-peer systems, quality of service and security management for distributed multimedia applications, and control procedures for optical networks.



**Michel Savoie** is the research program manager for the Broadband Applications and Optical Networks group of the Broadband Network Technologies Research Branch at the Communications Research Centre Canada (CRC), an Agency of Industry Canada. He maintains expertise in broadband systems and related technologies such as user-controlled lightpaths (UCLP); infrastructure as a service (IaaS); green ICT; application-oriented networking (AON); and advanced IP-, ATM-, and WDM-based optical networks in order to provide advice on important national initiatives and to demonstrate the application of CRC technologies in a real operational environment. He has managed two UCLP projects funded under CANARIE's directed research program and was involved with EUCALYPTUS, a Service-oriented Participatory Design Studio project funded under the CANARIE Intelligent Infrastructure Program (CIIP). He is also involved with PHOSPHORUS, a Lambda User Controlled Infrastructure for European Research integrated project funded by the European Commission under the IST 6th Framework; the Health Services Virtual Organization (HSVO) project under CANARIE's Network-Enabled Platforms program; and the High Performance Digital Media Network (HPDMnet) research initiative. CRC and its partners are currently engaged in the further development of the Argia/UCLP and the IaaS Framework software. Mr. Savoie holds a B.Sc. and M.Sc. in electrical engineering from the University of New Brunswick.