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Resource Criticality Analysis of Static Resource Allocations in WDM Networks

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***Abstract* — Various static resource allocation algorithms have been used in WDM networks to allocate resources such as wavelength channels, transmitters, receivers, and wavelength converters to a given set of static lightpath demands, based on certain design objectives. However, although optimized resource allocations can be obtained, it remains an open issue how to determine which resources are bottlenecks to achieve better performance. Existing static resource allocation algorithms do not explicitly measure the impact of a given resource on the design objective. In this paper, we propose such a measurement based on the Lagrangian Relaxation (LR) framework. We use the optimized values of Lagrange multipliers as a direct measurement of the criticality of resources. Such a quantitative measurement can be naturally acquired along with the optimization process to obtain the optimal solution (or a near optimal solution) to the static Routing and Wavelength Assignment (RWA) problem. Such a measurement helps to identify critical resources, and thus to decide the best way to add or reallocate resources.**

I. Introduction

Routing and Wavelength Assignment (RWA) algorithms are used to solve resource allocation problems in WDM networks. RWA algorithms have two flavours: static (also called offline) and dynamic (also called online). Static RWA algorithms use a given set of lightpath demands, and aim at providing a long-term plan for future traffic. In dynamic RWA algorithms, requests to establish or terminate lightpaths arrive dynamically. Previous studies have shown the value of using optimized paths that are computed offline to guide online path setups [1].

Existing RWA algorithms loosely or implicitly use the concept of resource criticality, without clearly defining or quantitatively measuring it. The most important strategy of almost all RWA heuristics is to avoid using "critical resources" and reserve them for future/other lightpath demands. However, there is no clear definition of what the criticality of resource means and how to calculate it. One must consider the complicated nature of interactions among competing demands and relationships between the different resources. Kodialam and Lakshman proposed a minimum interference routing algorithm for MPLS traffic engineering [2]. They consider that when some traffic demand is routed over a given link, the available flow values for one or more source-destination node pairs decrease. When these values reach a lower limit, the link is considered a critical link. However, to simplify the computation, they only consider one node pair at a time. Ho and Mouftah proposed asynchronous criticality avoidance routing to reduce the mutual interference between lightpaths launched by different node pairs [3]. Under the fixed alternative routing architecture, they measure the criticality of a link by the number of free wavelength channels on that link. When this number drops below a predefined threshold, that link is treated as a critical resource and other lightpath demands should avoid it. However, such a determination of resource criticality is very rough and does not explicitly reflect the impact of a given resource on the design objective.

In this paper, we propose a direct determination of the criticality of resources in static resource allocations of WDM networks. Within the Lagrangian Relaxation (LR) framework, the optimized values of Lagrange multipliers reflect the criticality of resources [4]. A method to compute these values for a given resource is presented. We demonstrate that the optimized Lagrange multipliers can successfully indentify critical resources in static RWA schemes, and thus help plan the network upgrading or resources reallocation to improve the design objective.

This paper is organized as follows: in Section II, we outline an integer linear programming formulation of the static RWA problem. Then in Section III, we explain how Lagrange multipliers can be used as a measure of resource criticality, followed by a computation method for the optimized Lagrange multipliers presented in Section IV. In Section V, we discuss examples that show how the optimized Lagrange multipliers are used to identify critical resources. We conclude this paper in Section VI.

II. An Integer Linear Programming Formulation of the Static RWA Problem

We use a mesh network topology with varying number of wavelength converters at different nodes, possibly zero. Wavelength converters are installed at a node in a share-per-node manner, which means any input or output port may use a wavelength converter if one is available. Our network model consists of N nodes interconnected by E fibres. Each fibre has W non-interfering Wavelength Channels (WCs). The fibre between nodes i and j is denoted by e_{ij}. The c^{th} WC on e_{ij} is denoted by w_{ijc} ($0 < c \leq W$). The set $\mathcal{E}$ represents all links in the network. Each link has a pair of fibres, one for each direction. The set $\mathcal{V}$ represents all the nodes in the network. A lightpath can be established between a source and destination node; such a lightpath is defined as a sequence of concatenated WCs. The WCs used by a lightpath on different links are allowed to use different wavelengths, if a wavelength converter is available at the intermediate node. Our model allows more than one lightpath being set up between a given node pair. s_{sdn} denotes the n^{th} lightpath demand between node pair (s,d). The set $\mathcal{L}$ represents all lightpath demands over the network.

We adopt a penalty-based formulation as our objective function as in [5]. We penalize the rejection of demands and the use of network resources. When a request is rejected, certain potential revenue is lost. Thus, the rejection penalty is the amount of its potential revenue. On the other hand, when a request is accepted, its resource consumption is added as a penalty in the objective function. The resource consumption penalty is the cost of resources that is used by the lightpath provisioned for the demand. Note that in the optimization process, our objective function also serves as the final performance evaluation measure, i.e., we optimize this criteria and also evaluate our results using the same criteria, unlike some heuristic algorithms, trying to optimize one criteria but then use another criteria as performance measure. Also note that, instead of using the objective function described in this paper, we could use the rejection ratio as the objective function in our optimization process; this would not include the consideration of fairness [6].

Our design objective is to minimize the penalty-based objective function *J*, i.e., $\min_{A,\Delta,\Phi}(J)$, where

$$J = \sum_{s_{sdn}\in\mathcal{L}}\left[(1-\alpha_{sdn})P_{sdn} + \alpha_{sdn}C_{sdn}\right] \qquad (1)$$

For each demand s_{sdn}, either the penalty of rejecting it (P_{sdn}), or the penalty of using resources (C_{sdn}) to set up a lightpath is added to the objective function (J), depending on s_{sdn}'s admission status α_{sdn}. α_{sdn} is zero, if s_{sdn} is rejected; and α_{sdn} is one, if s_{sdn} is admitted.

In addition to the design variables α_{sdn}, we introduce the design variables δ_{ijc}^{sdn}, representing the use of w_{ijc} by s_{sdn}, and the design variables ϕ_i^{sdn}, representing the use of a wavelength converter at node i by s_{sdn}. If w_{ijc} is used by s_{sdn}, δ_{ijc}^{sdn} equals one; otherwise, δ_{ijc}^{sdn} equals zero. We write A for the collection of all α_{sdn}, Δ for the collection of all δ_{ijc}^{sdn}, and Φ for the collection of all ϕ_i^{sdn}. We write Δ_{sdn} for the wavelength assignment of s_{sdn}. Now we may define the cost of resources C_{sdn} as the cost of using WCs and wavelength converters:

$$C_{sdn} = \sum_{e_{ij}\in\mathcal{E}}\sum_{0<c\le W} d_{ij}\delta_{ijc}^{sdn} + \sum_{i\in\mathcal{V}} o_i\phi_i^{sdn}, \qquad \forall s_{sdn}\in\mathcal{L} \qquad (2)$$

where d_{ij} is the cost of using w_{ijc}, o_i is the cost of using a wavelength converter at node i, ϕ_i^{sdn} is a 0-1 integer variable, representing the use of a wavelength converter at node i by s_{sdn}. If a wavelength converter is used by s_{sdn}, ϕ_i^{sdn} equals one; otherwise, ϕ_i^{sdn} equals zero.

The above static RWA problem must confine to the following constraints.

a) Lightpath continuity constraints:

If a demand is admitted, the lightpath assigned to it has to be continuous along a path between the source-destination pair. Since the assigned lightpath terminates at the two end nodes, we have

$$\sum_{j\in\mathcal{V}}\sum_{0<c\le W}\delta_{ijc}^{sdn} - \sum_{j\in\mathcal{V}}\sum_{0<c\le W}\delta_{jic}^{sdn} = \begin{cases}\alpha_{sdn} & \text{if } i=s\\ -\alpha_{sdn} & \text{if } i=d\\ 0 & \text{otherwise}\end{cases}, \qquad \forall s_{sdn}\in\mathcal{L} \qquad (3)$$

b) WC exclusive usage constraints:

$$\sum_{s_{sdn}\in\mathcal{L}}\delta_{ijc}^{sdn} \le 1, \qquad \forall e_{ij}\in\mathcal{E}, 0<c\le W \qquad (4)$$

These constraints mean that each WC can only be used by one lightpath.

c) Transmitter, receiver, and wavelength converter quantity constraints:

$$\sum_{d\in\mathcal{V}}\sum_{0<n\le N_{sd}}\alpha_{sdn} \le T_s, \qquad \forall s\in\mathcal{V} \qquad (5)$$

$$\sum_{s\in\mathcal{V}}\sum_{0<n\le N_{sd}}\alpha_{sdn} \le R_d, \qquad \forall d\in\mathcal{V} \qquad (6)$$

$$\sum_{s_{sdn}\in\mathcal{L}}\phi_i^{sdn} \le F_i, \qquad \forall i\in\mathcal{V} \qquad (7)$$

The number of lightpaths originating from or terminating at a node must be no more than the number of transmitters or receivers at the node. We assume that all transmitters and receivers operate at any wavelength. The number of transmitters at source node *s* is denoted by T_s. The number of receivers at destination node *d* is denoted by R_d. N_{sd} is the number of lightpath demands between (s,d). The number of used converters at a node must be no more than the number of the installed converters at the node. The number of wavelength converters at node *i* is denoted by F_i.

d) Wavelength conversion constraints:

$$\phi_j^{sdn} = \begin{cases}1 & \text{if } \exists m,k\in\mathcal{V} \text{ and } b\ne a, \delta_{mja}^{sdn} = \delta_{jkb}^{sdn} = 1\\ 0 & \text{otherwise}\end{cases} \qquad \forall j\in\mathcal{V} \qquad (8)$$

A wavelength converter at an intermediate node *j* is used only when different wavelengths are assigned to s_{sdn} for the incoming and outgoing signals at this node.

III. LAGRANGE MULTIPLIERS AS A DIRECT MEASUREMENT OF THE RESOURCE CRITICALITY

We use the LR framework to derive a Dual Problem (DP) from the primal problem $\min_{A,\Delta}(J)$, by relaxing selected constraints. Lagrange multipliers ξ_{ijc}, π_s, θ_d, and λ_i are introduced in association with the constraints that represent resource limitations, that is, the WC exclusive usage constraints in (4), transmitter, receiver and wavelength converter quantity constraints in (5-7), respectively. The Lagrangian function *L* is defined as [6]:

$$L(A,\Delta,\Phi,\xi,\pi,\theta,\lambda) = J(A,\Delta,\Phi) + \sum_{e_{ij}\in\mathcal{E}}\sum_{0<c\le W}\xi_{ijc}\left(\sum_{s_{sdn}\in\mathcal{L}}\delta_{ijc}^{sdn} - 1\right) + \sum_{s\in\mathcal{V}}\pi_s\left(\sum_{d\in\mathcal{V}}\sum_{0<n\le N_{sd}}\alpha_{sdn} - T_s\right) + \sum_{d\in\mathcal{V}}\theta_d\left(\sum_{s\in\mathcal{V}}\sum_{0<n\le N_{sd}}\alpha_{sdn} - R_d\right) + \sum_{i\in\mathcal{V}}\lambda_i\left(\sum_{s_{sdn}\in\mathcal{L}}\phi_i^{sdn} - F_i\right), \qquad (9)$$

where $\xi,\pi,\theta,\lambda \ge 0$. The vectors of Lagrange multipliers (ξ_{ijc}),

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(π_s), (θ_d), and (λ_i) are denoted as ξ, π, θ and λ, respectively.

Define the dual function $q(\xi,\pi,\theta,\lambda)$ as the infimum of L as a function of the Lagrange multipliers and design variables. q^* denotes the optimal value of the DP. When the dual function reaches its maximum and the Lagrangian function reaches its minimum, we get the following bound for the Lagrangian DP:

$$q^* \le \min_{\Lambda,\Delta,\Phi}(J) \qquad (10)$$

subject to the constraints in (3) and (8).

We use the optimized values of Lagrange multipliers as a direct measurement of the criticality of resources. The optimized values of Lagrange multipliers represent the sensitivity of the objective function J with respect to the level of a given resource. For simplicity, we use the terminology "optimized Lagrange multipliers" to refer to the near-optimal values of Lagrange multipliers. For an optimized Lagrange multiplier M_R^* (i.e. the optimized values of ξ_{ijc}, π_s, θ_d and λ_i) corresponding to a given resource R with a continuous resource level, we have

$$M_R^* = -\frac{dJ^*}{dR} \qquad (11)$$

For a resource with discrete levels, the corresponding optimized Lagrange multiplier is only an estimate of the sensitivity of the objective function J with respect to the level of the resource.

$$M_R^* \cong -\frac{\Delta J^*}{\Delta R} \qquad (12)$$

When the optimized Lagrange multiplier M_R^* for a given resource R is known, the impact from any change of the resource on the design objective can be estimated. For example, when a small amount (ΔR) of a critical resource R is added into the network, the improvement of the design objective can be estimated as $M_R^* \times \Delta R$. It should be noted that such an estimation is very rough and only applies to minor resource changes. In general, because the propagation effect is very complicated, the improvement of the design objective needs to be re-computed by solving a new optimization problem.

The optimized Lagrange multipliers for different resources can be used as a quantitative measurement for the relative significance of the resources. When we add resources with high optimized Lagrange multipliers into the network, the improvement of the design objective is greater than adding resources with low optimized Lagrange multipliers. In this way, the optimized Lagrange multipliers help to identify the bottleneck resources for performance improvements. Thus, new resources can be more efficiently added into the network, or the existing resources can be reallocated to a more efficient configuration.

IV. Computation of Optimized Lagrange Multipliers

To compute optimized Lagrange multipliers, we need to solve the Lagrangian DP. The key of solving the Lagrangian DP is to derive independent sub-problems, where the optimal solutions to the sub-problems can be computed. By using the fact that $\delta_{ijc}^{sdn} = \alpha_{sdn}\delta_{ijc}^{sdn}$, $\phi_i^{sdn} = \alpha_{sdn}\phi_i^{sdn}$, and removing the terms that are independent of the decision variables, the Lagrangian DP becomes:

$$\min_{\Lambda,\Delta,\Phi}\left\{\sum_{s_{sdn}\in\mathcal{L}}\left[(1-\alpha_{sdn})P_{sdn} + \alpha_{sdn}\left(\sum_{e_{ij}\in\mathcal{E}}\sum_{0<c\le W}\delta_{ijc}^{sdn}\left(\xi_{ijc}+d_{ij}\right) + \sum_{i\in\mathcal{V}}\phi_i^{sdn}\left(\lambda_i+o_i\right) + \pi_s + \theta_d\right)\right]\right\} \qquad (13)$$

The resource allocation to each lightpath becomes independent, because the resource usage constraints in (4-7) are relaxed. The complex competition among lightpaths for shared resources does not need to be considered when we allocate resources to each lightpath. Then each sub-problem corresponds to the decision of acceptance or rejection of a single lightpath demand, and to the associated RWA problem for each accepted lightpath demand. The number of independent sub-problems is the total number of lightpath demands. The optimal value of the relaxed problem is the summation of the optimal values of all lightpath-level sub-problems (denoted SP_{sdn} for the sub-problem that corresponds to s_{sdn}):

$$\sum_{s_{sdn}\in\mathcal{L}}\min_{\alpha_{sdn},\Delta_{sdn},\Phi_{sdn}}\left[(1-\alpha_{sdn})P_{sdn} + \alpha_{sdn}\left(\sum_{e_{ij}\in\mathcal{E}}\sum_{0<c\le W}\delta_{ijc}^{sdn}\left(\xi_{ijc}+d_{ij}\right) + \sum_{i\in\mathcal{V}}\phi_i^{sdn}\left(\lambda_i+o_i\right) + \pi_s + \theta_d\right)\right] \qquad (14)$$

We use the Lagrangian Relaxation and Subgradient Method (LRSM) to compute optimized Lagrange multipliers. At the same time, a solution to the original problem (i.e., the primal problem) is derived from the solutions to individual sub-problems. The LRSM solves the Lagrangian DP iteratively by adjusting the values of the Lagrange multipliers. It is illustrated in Figure 1. When the iteration converges, the optimized Lagrange multipliers are obtained. In each iterative round, a solution to the Lagrangian DP is mapped to a solution of the primal problem by applying a heuristic algorithm. In addition to the built-in nature of attempting to respect the relaxed constraints in solving the Lagrangian DP, the heuristic algorithm forces the relaxed constraints to be respected.

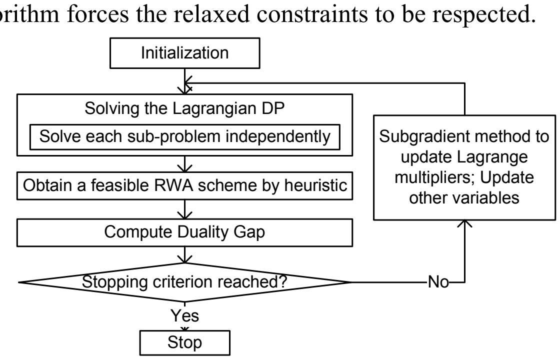

Figure 1. Schematic depiction of the overall algorithm

Each sub-problem SP_{sdn} in (14) can be solved in two steps: lightpath routing, and acceptance/rejection decision. The first step is to solve the lightpath routing problem:

$$D_{sdn} = \min_{\Delta_{sdn}}\left\{\sum_{e_{ij}\in\mathcal{E}}\sum_{0<c\le W}\delta_{ijc}^{sdn}\left(\xi_{ijc}+d_{ij}\right) + \sum_{i\in\mathcal{V}}\phi_i^{sdn}\left(\lambda_i+o_i\right)\right\}, \qquad (15)$$

subject to constraints (3) and (8) for s_{sdn}. We assign an auxiliary cost $(\xi_{ijc} + d_{ij})$ to w_{ijc}. The optimal solution is computed by using the modified minimum cost semi-lightpath algorithm in [6].

The second step is to solve the decision problem:

$$\min_{\alpha_{sdn}} [(1-\alpha_{sdn})P_{sdn} + \alpha_{sdn}(D_{sdn} + \pi_s + \theta_d)] \quad (16)$$

If P_{sdn} is greater than $(D_{sdn} + \pi_s + \theta_d)$, then rejecting s_{sdn} is more beneficial to the objective. On the contrary, if P_{sdn} is smaller, then we accept s_{sdn} by assigning one to the design variable α_{sdn}. A tie is broken arbitrarily.

We use the subgradient method to search optimized Lagrange multipliers. The Lagrange multiplier vector $z = (\xi_{ijc}, \pi_s, \theta_d, \lambda_i)$ is updated towards the direction of its subgradient:

$$z^{(h+1)} = z^{(h)} + \alpha^{(h)} g(z^{(h)}), \quad (17)$$

where $z^{(h)}$ denotes the value of vector z obtained at the h^{th} iteration, and $\alpha^{(h)}$ denote the step size in the h^{th} iteration. The vector $g(z)$ is the subgradient of the dual function q with respect to z, i.e. $g(z) = (g_{ijc}(\xi), g_s(\pi), g_d(\theta), g_i(\lambda))$.

$$g_{ijc}(\xi) = \sum_{s_{sdn} \in \mathcal{L}} \delta_{ijc}^{sdn} - 1, \qquad \forall e_{ij} \in \mathcal{E}, 0 < c \le W \quad (18)$$

$$g_s(\pi) = \sum_{d \in \mathcal{V}} \sum_{0 < n \le N_{sd}} \alpha_{sdn} - T_s, \qquad \forall s \in \mathcal{V} \quad (19)$$

$$g_d(\theta) = \sum_{s \in \mathcal{V}} \sum_{0 < n \le N_{sd}} \alpha_{sdn} - R_d, \qquad \forall d \in \mathcal{V} \quad (20)$$

$$g_i(\lambda) = \sum_{s_{sdn} \in \mathcal{L}} \phi_i^{sdn} - F_i, \qquad \forall i \in \mathcal{V}. \quad (21)$$

V. EXAMPLES

In the first example, we use optimized Lagrange multipliers to quantify the criticality of the WCs, then add or reallocate WCs to the critical links, and evaluate the improvement of the design objective. The purpose of this example is to demonstrate that for a given set of static lightpath demands over a given mesh WDM network topology, the optimized Lagrange multipliers can effectively identify the bottleneck resources. Then we verify that the identified resources are actually bottlenecks for performance improvement by performing two simulations: (a) adding resources just to the identified bottlenecks, and (b) reallocating resources from non-bottlenecks to the identified bottlenecks. Both experiments show performance improvements. In this example, we use NSFNET shown in Figure 2. The static lightpath demands are shown in Table 1, where the horizontal index of the matrix is the source node of a lightpath demand, while the vertical index is the destination node.

TABLE 1. LIGHTPATH DEMAND MATRIX

0	1	3	1	3	1	3	0	2	0	3	2	0	3
0	0	0	2	0	2	1	0	1	0	1	0	0	3
3	2	0	3	0	1	2	3	2	3	1	2	2	0
3	1	0	0	1	1	2	3	2	2	1	2	0	3
1	3	0	2	0	1	0	2	0	3	0	1	1	3
1	2	1	3	2	0	1	3	3	1	0	1	0	2
2	2	3	1	3	3	0	0	3	1	2	0	3	3
0	1	2	0	1	0	1	0	0	1	0	0	2	0
3	0	1	3	3	3	1	0	0	2	1	1	1	2
0	0	0	1	2	0	2	0	1	0	1	0	0	3
1	0	0	2	0	3	0	1	0	3	0	3	0	3
2	3	1	1	3	2	3	2	2	2	2	0	1	3
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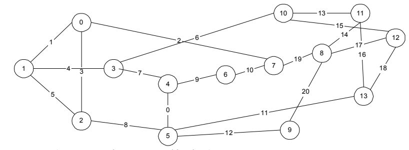

Figure 2. NSFNET (14 nodes, 21 links)

We compute optimized Lagrange multipliers for all WCs, then use the average value for the WCs on the same link as a measurement for the link. The WCs using different wavelengths on the same link have equal impact on the performance; only the number of WCs on a link matters. We assume the revenue for each lightpath is 1000 (P_{sdn}=1000), the cost of each WC is 250 (d_{ij}=250), the cost of each wavelength converter is zero, and many wavelength converters are available at each node. The number of transmitters and receivers at each node is set to 28 ($T_i = R_i$=28). Initially, we set the number of wavelengths on each link to 16 (W=16). Table 2 shows the average optimized Lagrange multipliers for all links. The design objective function value that corresponds to this resource allocation scheme is 160924 with a lower bound being 158849 as shown by Case 1 in Figure 3.

TABLE 2. THE AVERAGE OPTIMIZED LAGRANGE MULTIPLIERS FOR ALL LINKS

Link Number	Average Optimized Lagrange Multipliers (Direction 1)	Average Optimized Lagrange Multipliers (Direction 2)
0	**114.0**	**120.8**
1	0.1	0.2
2	0.4	52.1
3	0.1	0.4
4	0.4	0.4
5	0	0.2
6	**179.7**	**97.1**
7	0.4	10.4
8	26.4	0.5
9	0.2	0.2
10	52.2	26.2
11	**163.2**	**143.1**
12	14.9	0.5
13	0.6	0.2
14	0.3	0.6
15	0.3	0.4
16	0.3	0.2
17	0.4	0.4
18	0.3	0.2
19	**68.6**	**108.7**
20	0.6	0.1

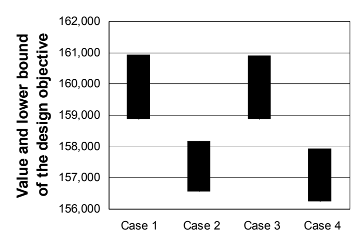

Figure 3. Achieved value and a lower bound of the design objective

The design objective under different resource configurations and/or amount is compared (see Figure 3). Note that the top of a bar represents the achieved value of the design objective corresponding to a feasible static RWA scheme obtained by the LRSM and the heuristics; the bottom of a bar represents a computed lower bound of the design objective. The truly optimal solution leads to a value of the design objective within the range of the bar. Since the design objective is to minimize the penalty, Cases 2 and 4 outperform

Cases 1 and 3. In Case 2, we add 2 WCs to the 4 most critical links identified in Table 2, i.e., links 0, 6, 11 and 19. As a comparison, in Case 3, we add 2 WCs to 4 randomly selected non-critical links, i.e., 1, 5, 9 and 18. In Case 4, instead of adding resources, we reallocate 2 WCs from non-critical links to the 4 most critical links, i.e., from links 1, 5, 9 and 18 to links 0, 6, 11 and 19.

In the second example, we use optimized Lagrange multipliers to identify the bottleneck locations of transmitters and receivers. The network topology and lightpath demands are the same as above (see Figure 2 and Table 1). We compute optimized Lagrange multipliers for transmitters and receivers at all nodes; results are shown in Table 3. Unlike the first example, where the transmitters and receivers are abundant, in the second example, we set the number of transmitters and receivers at all nodes to 20. In this way, transmitters and receivers are critical resources. Other parameters remain the same as in the first example. The design objective function value is 166375 with a lower bound being 166167 as shown by Case 1 in Figure 4. The optimized Lagrange multipliers help to identify the 4 bottleneck locations of receivers (nodes 5, 6, 10 and 13), and the 4 bottleneck locations of transmitters (nodes 4, 6, 8 and 11). We add one more transmitter or receiver at each of their bottleneck locations. The achieved value of the design objective function is improved, shown as Case 2 in Figure 4. As a comparison, we randomly add the same number of transmitters (for example, at nodes 1, 7, 9 and 10) and receivers (at nodes 0, 2, 11 and 12). The achieved value of the design objective function is not improved at all, as shown by Case 3 in Figure 4. In the last simulation, we reallocate transmitters from 4 non-bottleneck locations (nodes 1, 7, 9 and 10) to bottleneck locations (nodes 4, 6, 8 and 11). Meanwhile, we reallocate receivers from 4 non-bottleneck locations (nodes 0, 2, 11 and 12) to bottleneck locations (nodes 5, 6, 10, and 13). The improvement on the design objective is the same as adding new transmitters and receivers, as shown by Case 4 in Figure 4. This indicates that with the help of the optimized Lagrange multipliers, even without adding new resources, the design objective can be improved by reallocating existing resources.

TABLE 3. THE OPTIMIZED LAGRANGE MULTIPLIERS FOR TRANSMITTERS AND RECEIVERS AT ALL NODES

Node Number	Optimized Lagrange Multipliers for Transmitters	Optimized Lagrange Multipliers for Receivers
0	0	0
1	0	0
2	89.1	0
3	57.7	4.0
4	**246.1**	0
5	41.9	**85.0**
6	**248.7**	**158.1**
7	0	0
8	**245.1**	0
9	0	0
10	0	**249.2**
11	**247.3**	0
12	0	0
13	245.1	**239.2**

In the third example, we use optimized Lagrange multipliers to quantify the criticality of wavelength converters. The same network topology and lightpath demands are used as in the previous two examples. We compute optimized Lagrange multipliers for wavelength converters at all nodes (results are shown in Table 4). Unlike the previous two examples, where the wavelength converters are abundant, in the third example, we set their number at all nodes to 1. The results show that wavelength converters in a static RWA are not critical. Their contribution to the design objective is very minimal. This is consistent with other studies [7].

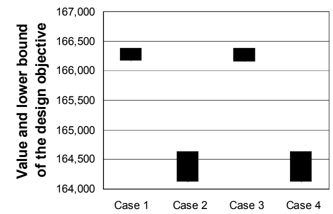

Figure 4. Achieved value and a lower bound of the design objective

TABLE 4. THE OPTIMIZED LAGRANGE MULTIPLIERS FOR WAVELENGTH CONVERTERS AT ALL NODES

Node number	Optimized Lagrangian Multipliers	Node number	Optimized Lagrangian Multipliers
0	0.36	7	0.36
1	0.09	8	0.35
2	0	9	0
3	0	10	0
4	0	11	0
5	0	12	0
6	0	13	0

VI. CONCLUSIONS

We proposed the use of optimized Lagrange multipliers as a direct measure of resource criticality in the context of the Routing and Wavelength Assignment (RWA) problem for WDM networks. It is well known that the optimized Lagrange multipliers reflect the impact of resources on the design objective. We showed how the Lagrangian Relaxation and Subgradient Method (LRSM) can be used to compute the optimized Lagrange multipliers for the RWA problem. The proposed determination of resource criticality and its computation method can be applied to a wide range of static RWA problems that are formulated as ILP problems. Simulation results indicate that the optimized Lagrange multipliers successfully indentify critical resources and thus help to plan network reconfigurations by adding new resources or re-allocating existing resources.

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